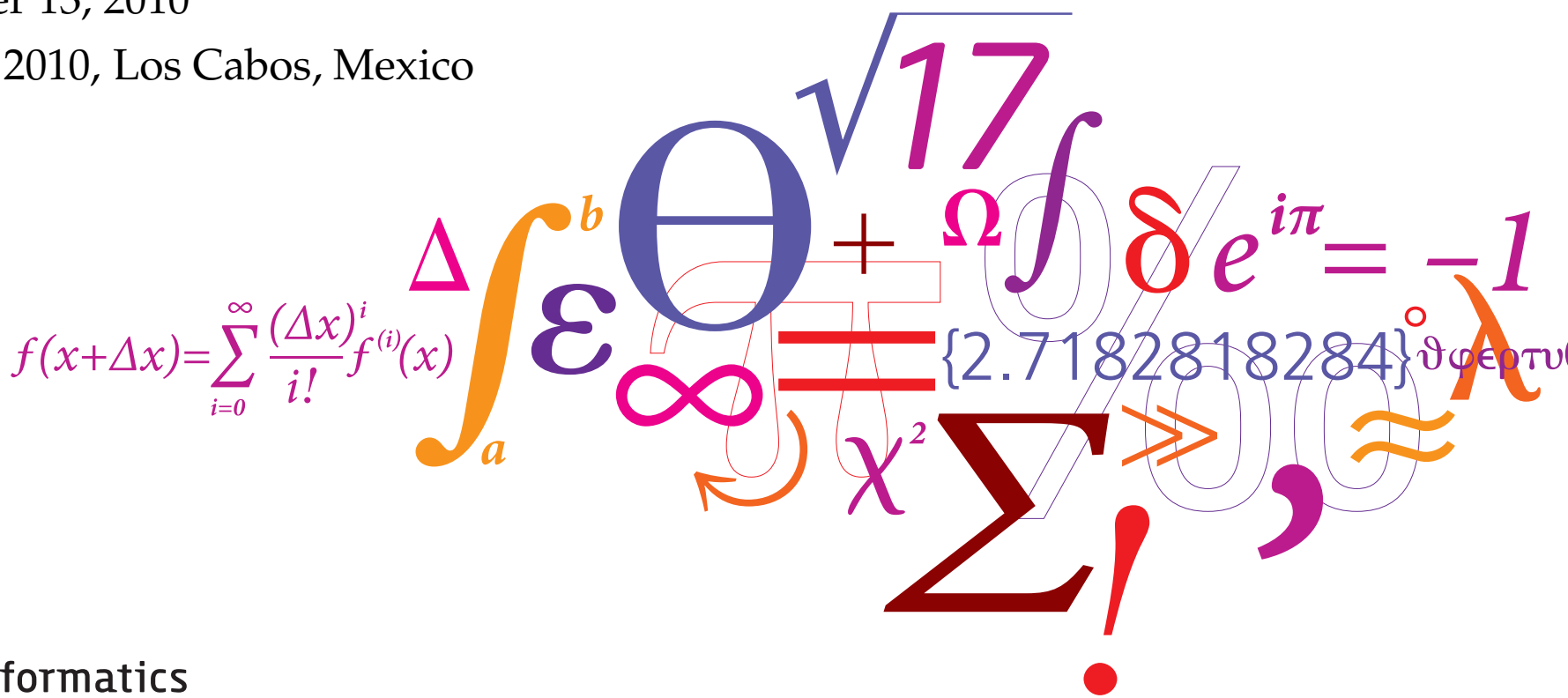
String Matching with Variable Length Gaps

By Philip Bille, Inge Li Gørtz, Hjalte Wedel Vildhøj and David Kofoed Wind

Presented by Hjalte Wedel Vildhøj

October 13, 2010

SPIRE 2010, Los Cabos, Mexico



DTU Informatics
Department of Informatics and Mathematical Modelling

The Variable Length Gap Problem

Given some string $T \in \Sigma^+$ and a *variable length gap pattern*

$$P = P_1 \cdot \ g\{a_1, b_1\} \cdot P_2 \cdot g\{a_2, b_2\} \cdots g\{a_{k-1}, b_{k-1}\} \cdot P_k \ .$$

Find the *end positions* for all occurrences of P in T.

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Example: $P = \mathrm{A} \cdot g\{6, 7\} \cdot \mathrm{CC} \cdot g\{2, 6\} \cdot \mathrm{GT}$

$T =$ ATCGGCTCCAGACCAGTACCCGTTCCGTGGT

Solution: $\{\}$

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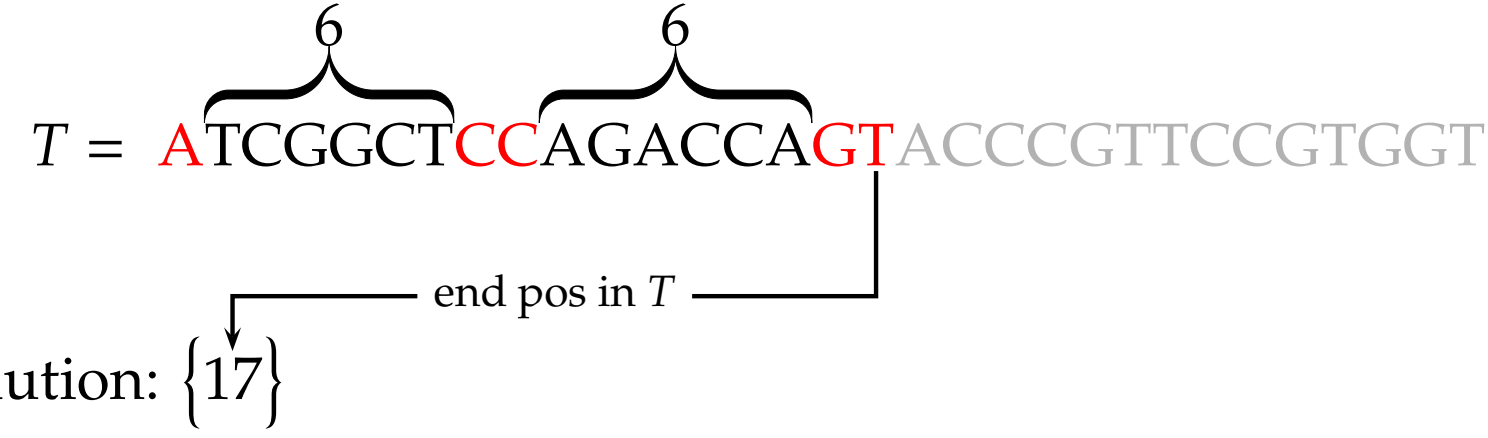

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$T =$ ATCGGCTCCA$\overbrace{\text{GACCAGTA}}^{8}CC\overbrace{\text{CGTTCC}}^{6}$GTGGT

Solution: $\{17\}$

Not a valid match!

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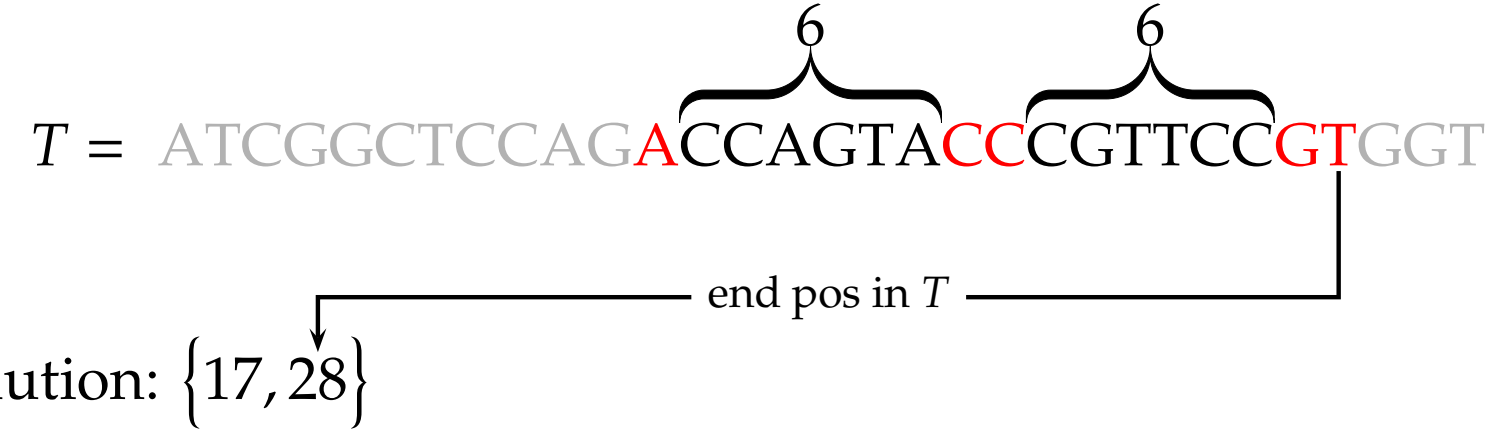

Solution: $\{17, 28\}$

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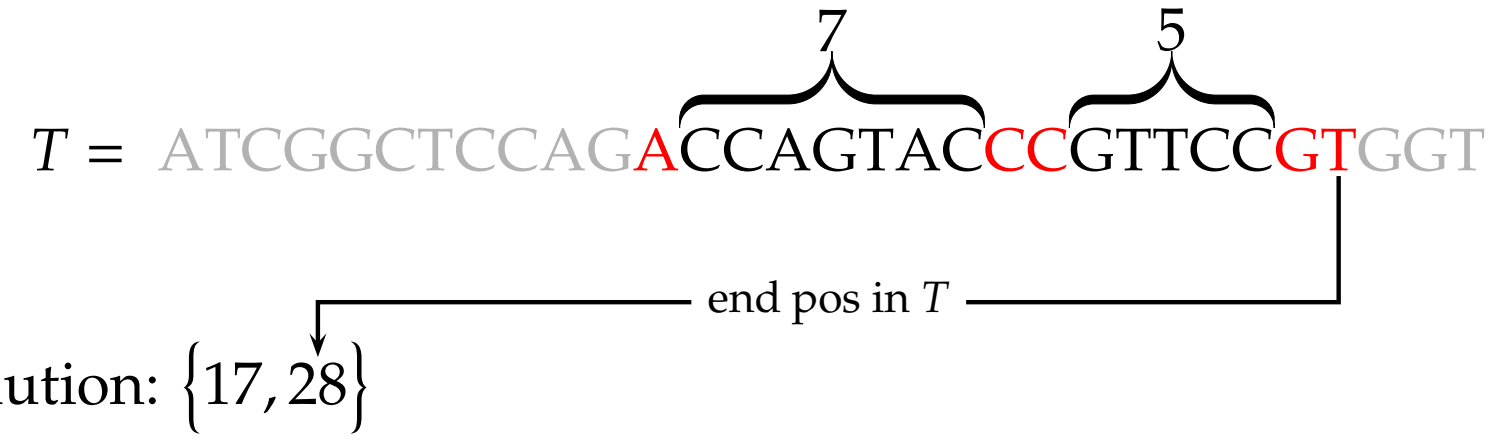

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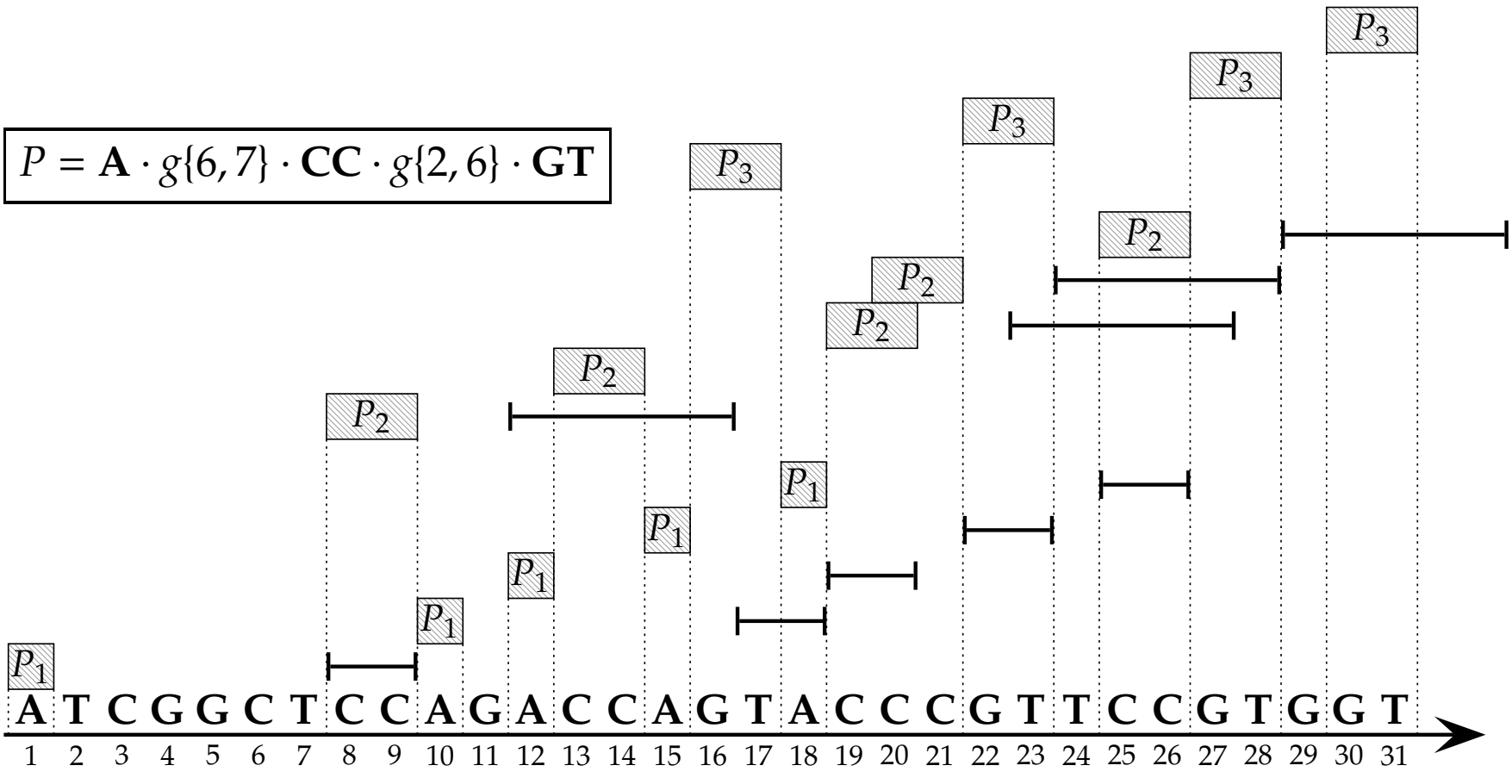

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Parameters

$n = |T|$

$\alpha = \#\text{ occ. of } P_1, P_2, \ldots, P_k \text{ in } T$

$m = \sum_{i=1}^{k} |P_i|$

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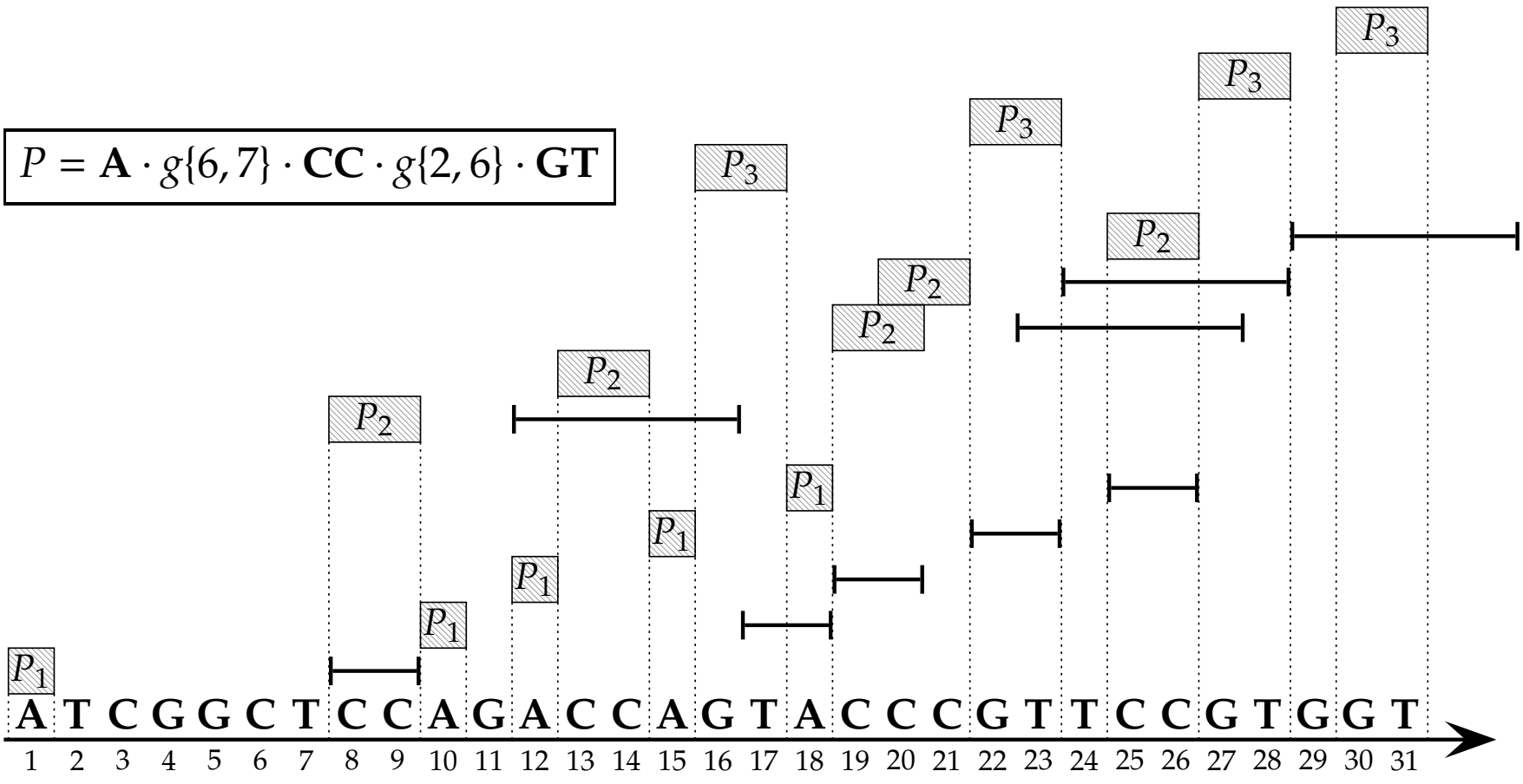

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Known Upper Bounds

By	*Time*	*Space*
Bille & Thorup[1]	$O\left(n(k\frac{\log w}{w} + \log k) + m \log m + A\right)$	$O(m + A)$
Morgante et al.[2]	$O((n + m) \log k + \alpha)$	$O(m + \alpha)$

[1] P. Bille and M. Thorup. Regular expression matching with multi-strings and intervals. In *Proc. 21st SODA, 2010*

[2] M. Morgante, A. Policriti, N. Vitacolonna, and A. Zuccolo. Structured motifs search. *J. Comput. Bio.*, 12(8):1065-1082, 2005

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Can you get the best of both?

Illustrating the Algorithm

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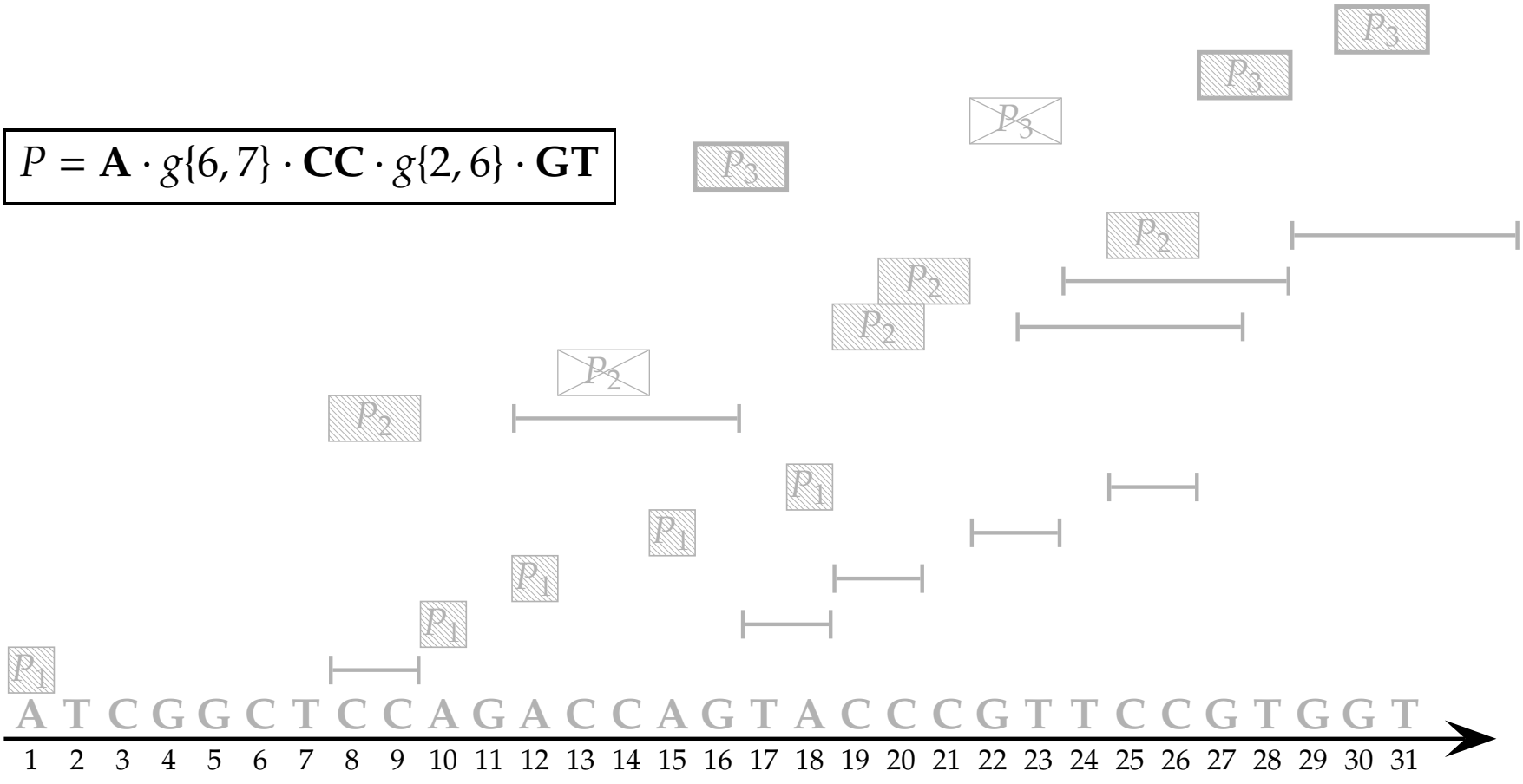

L_2

L_3

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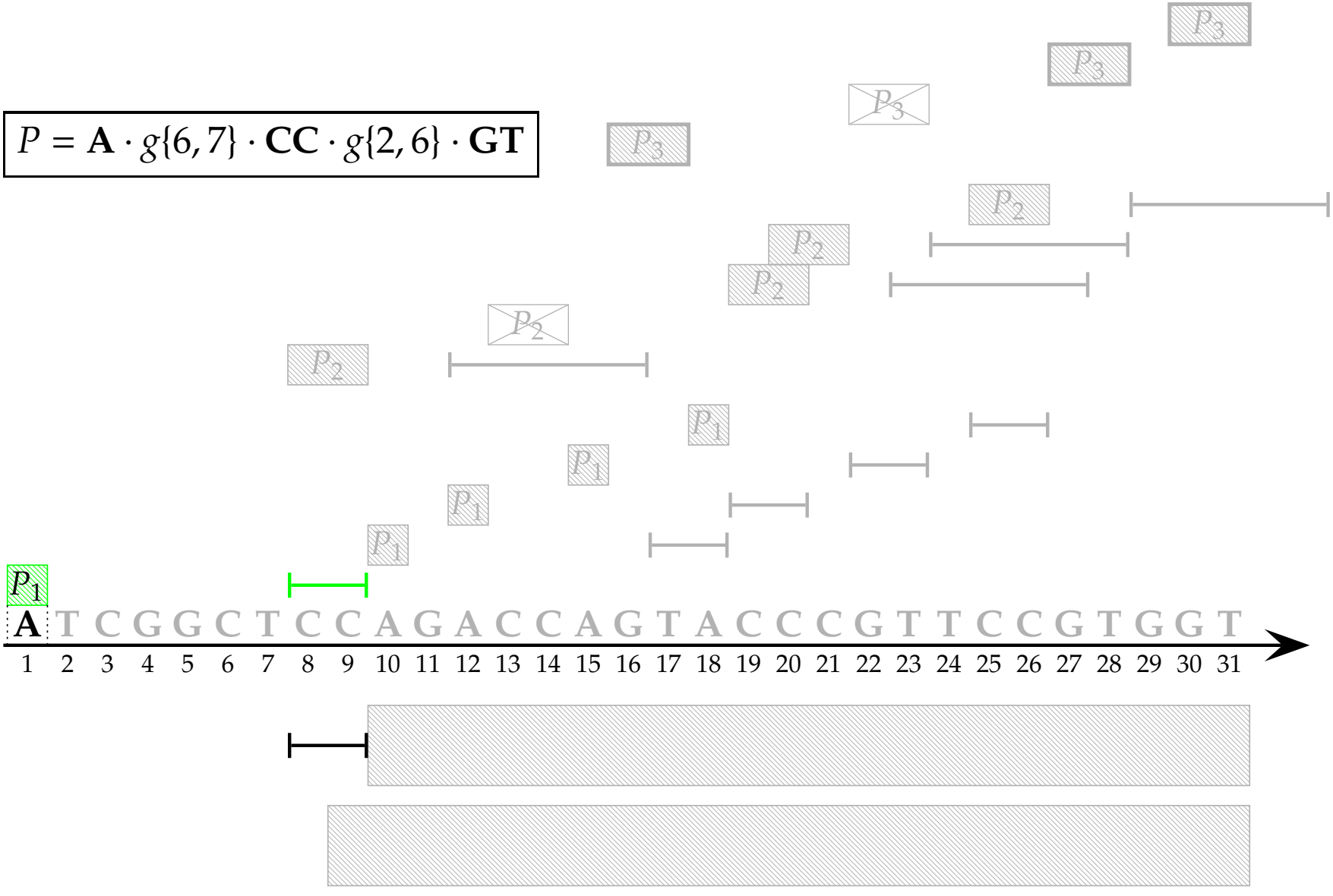

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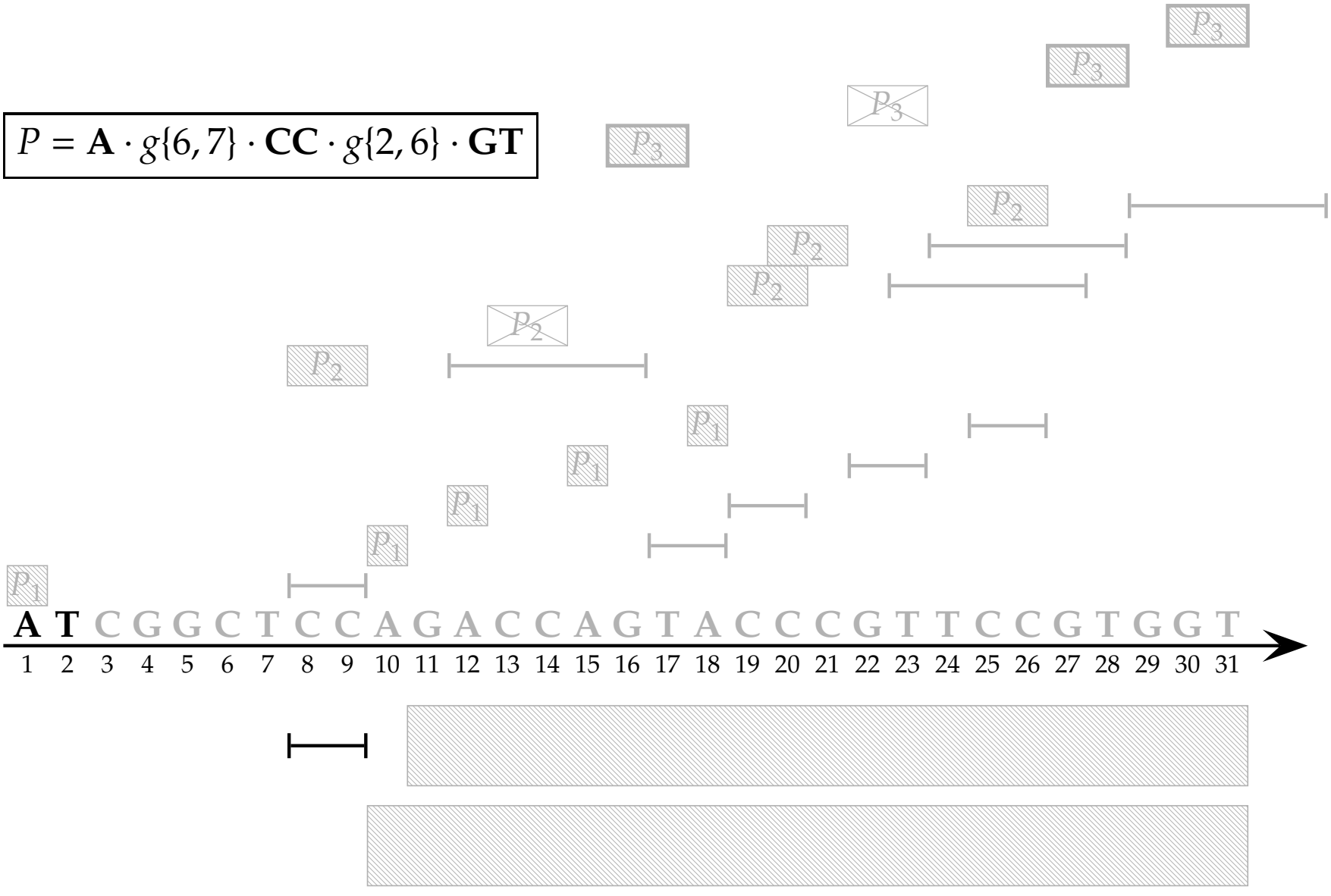

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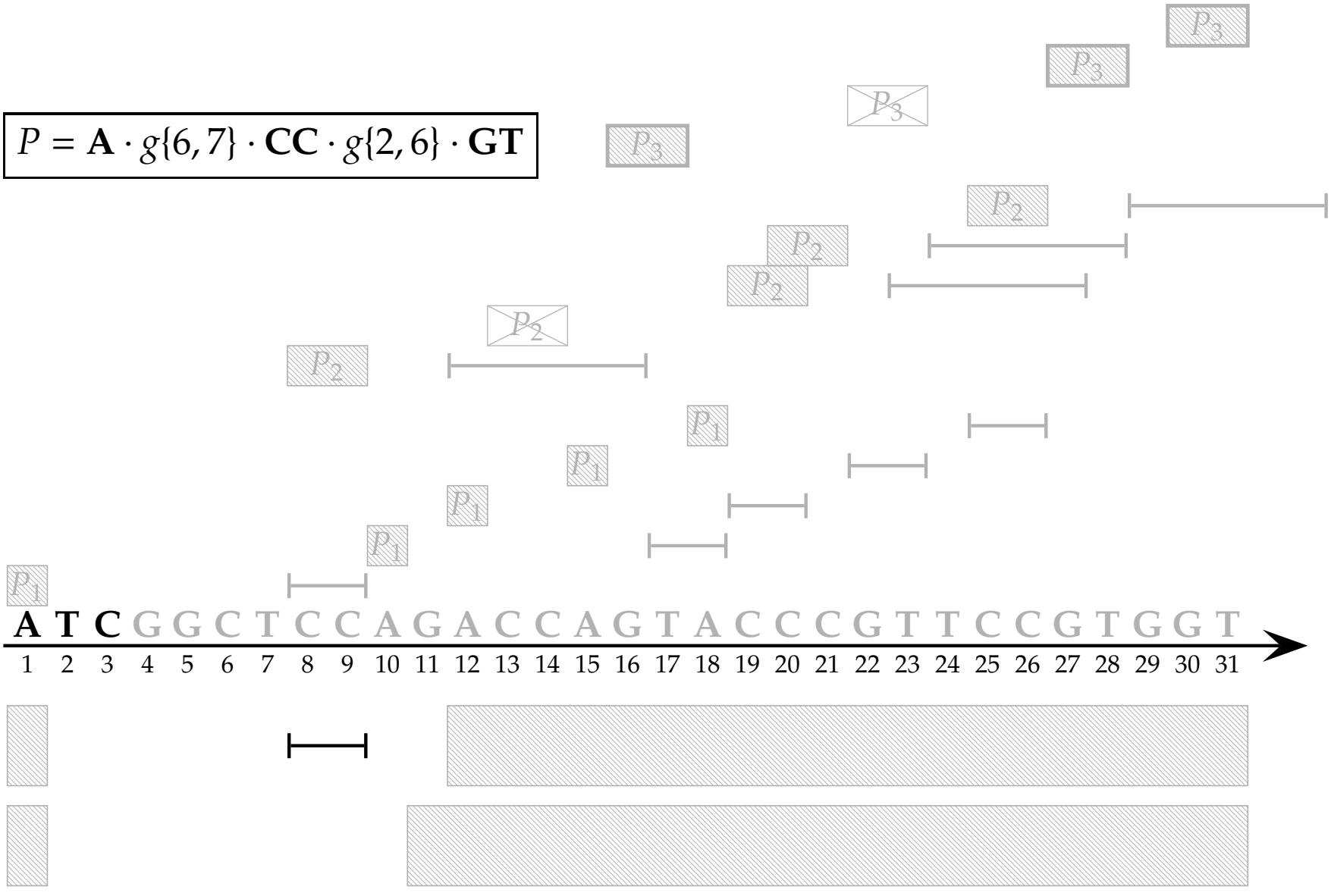

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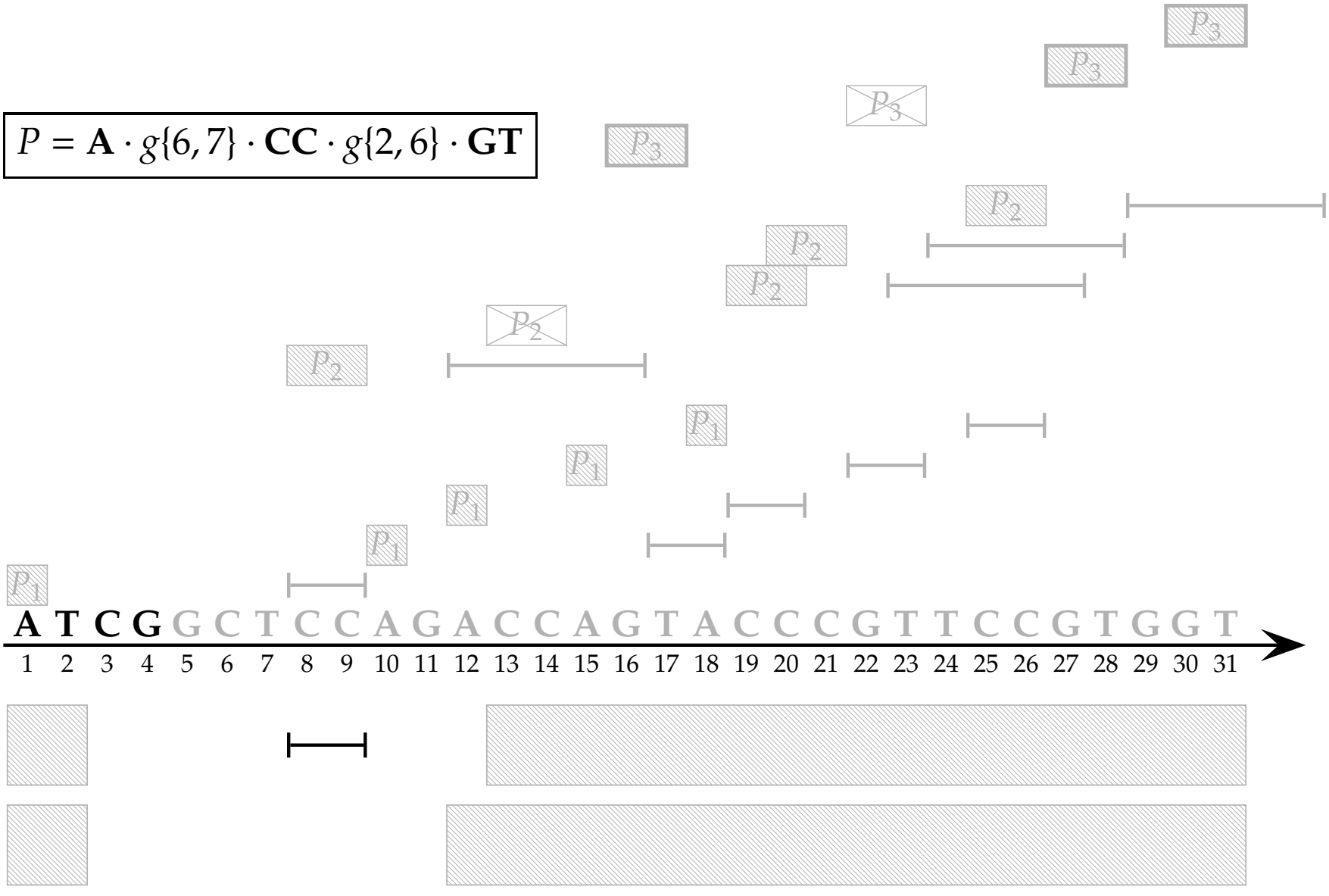

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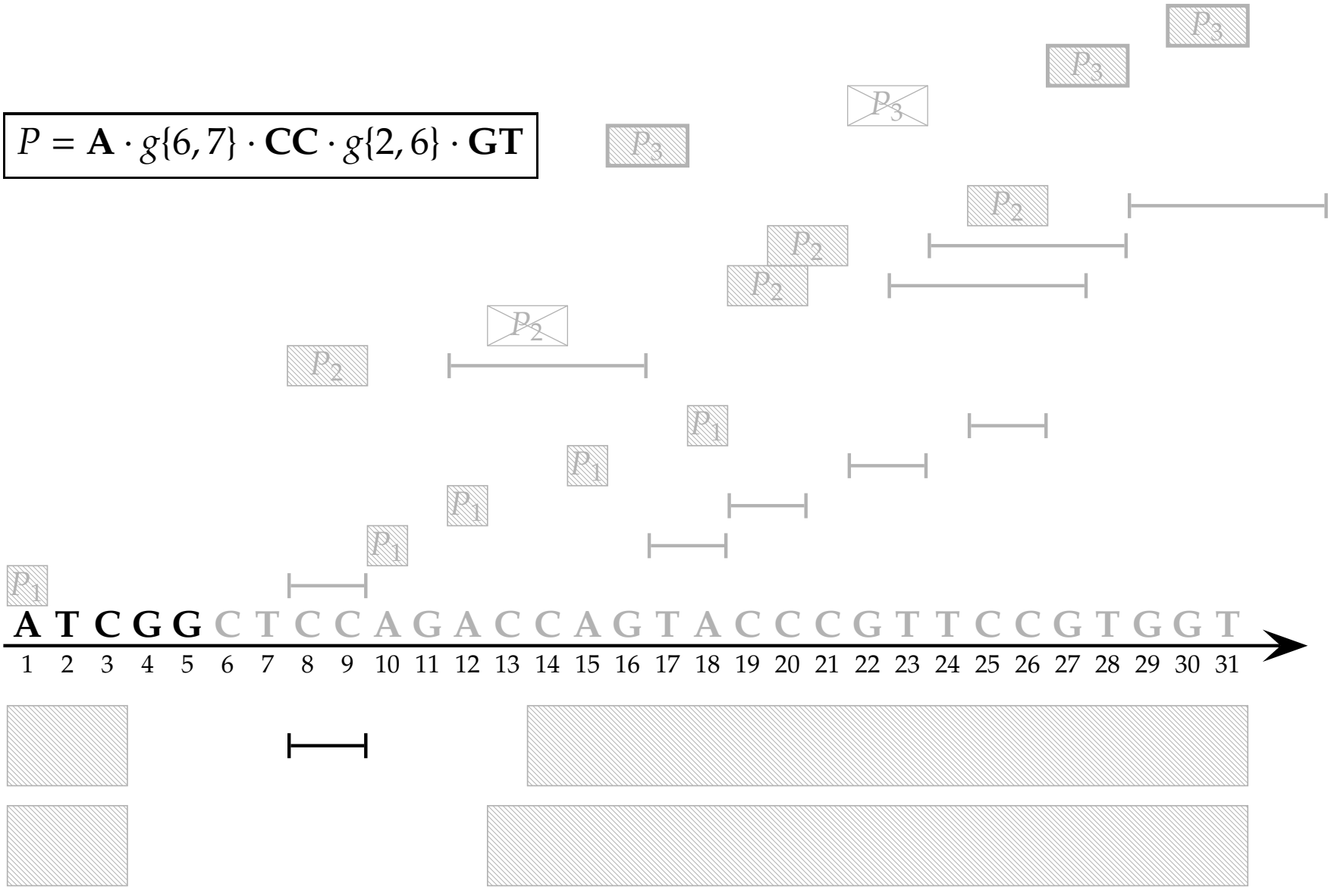

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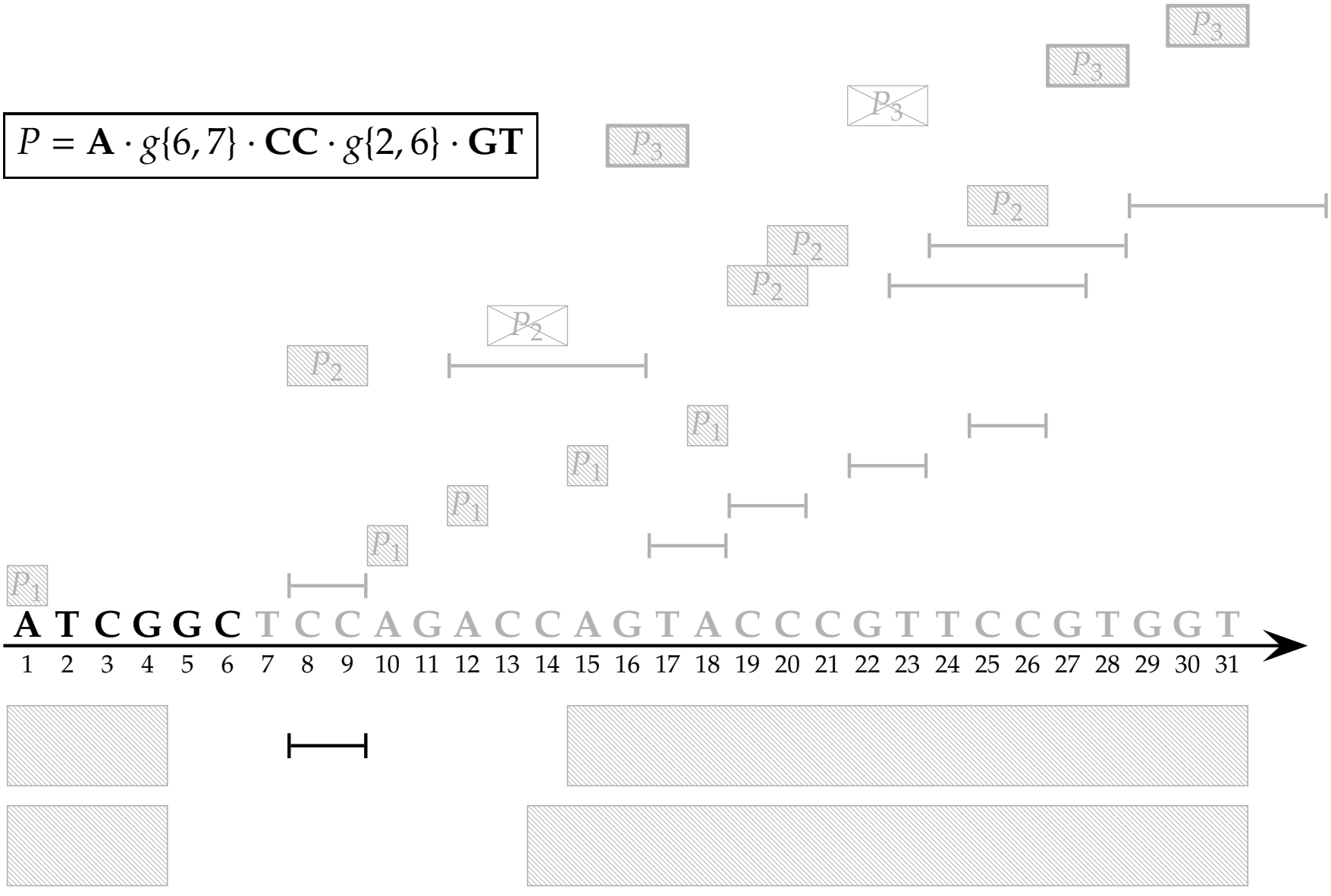

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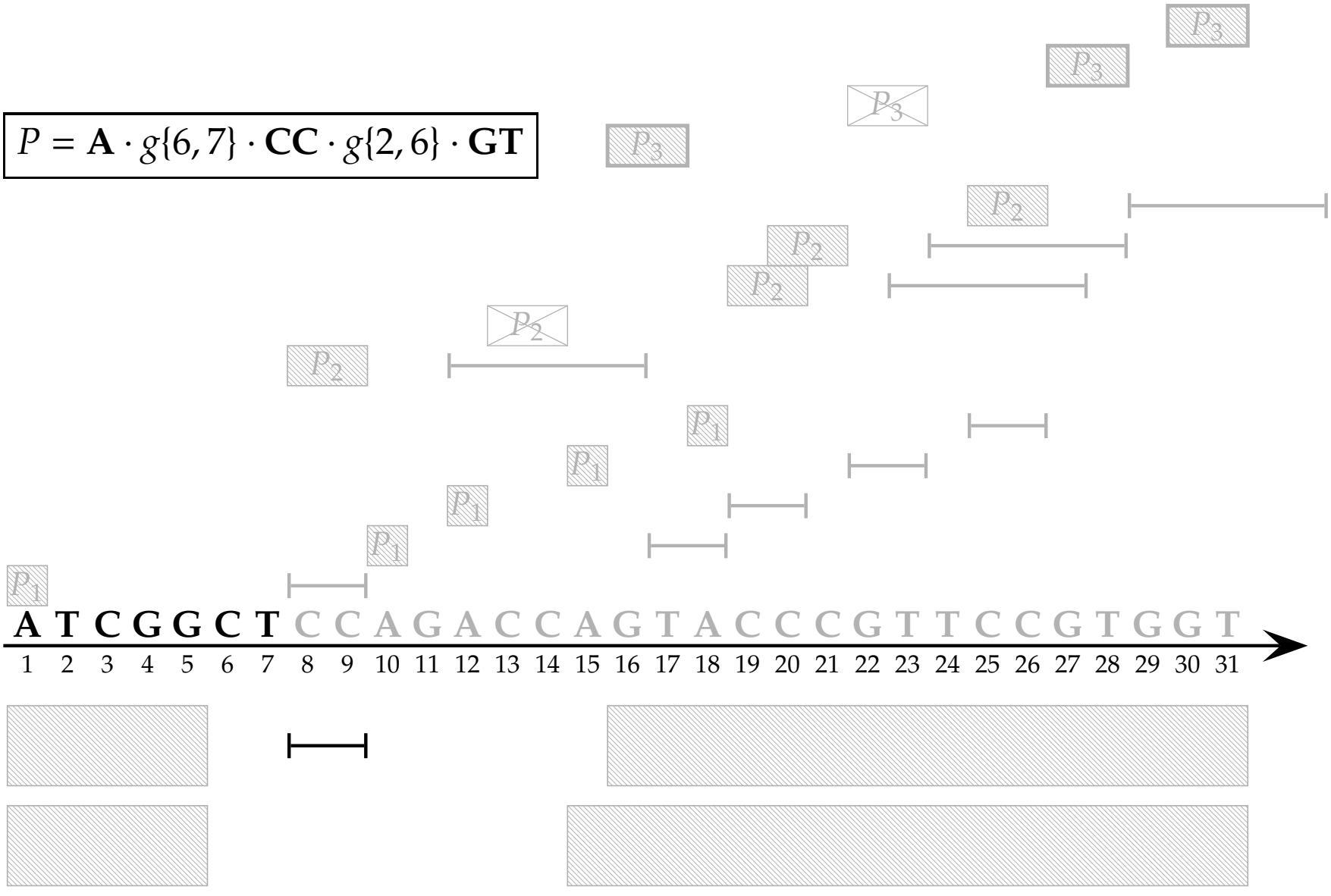

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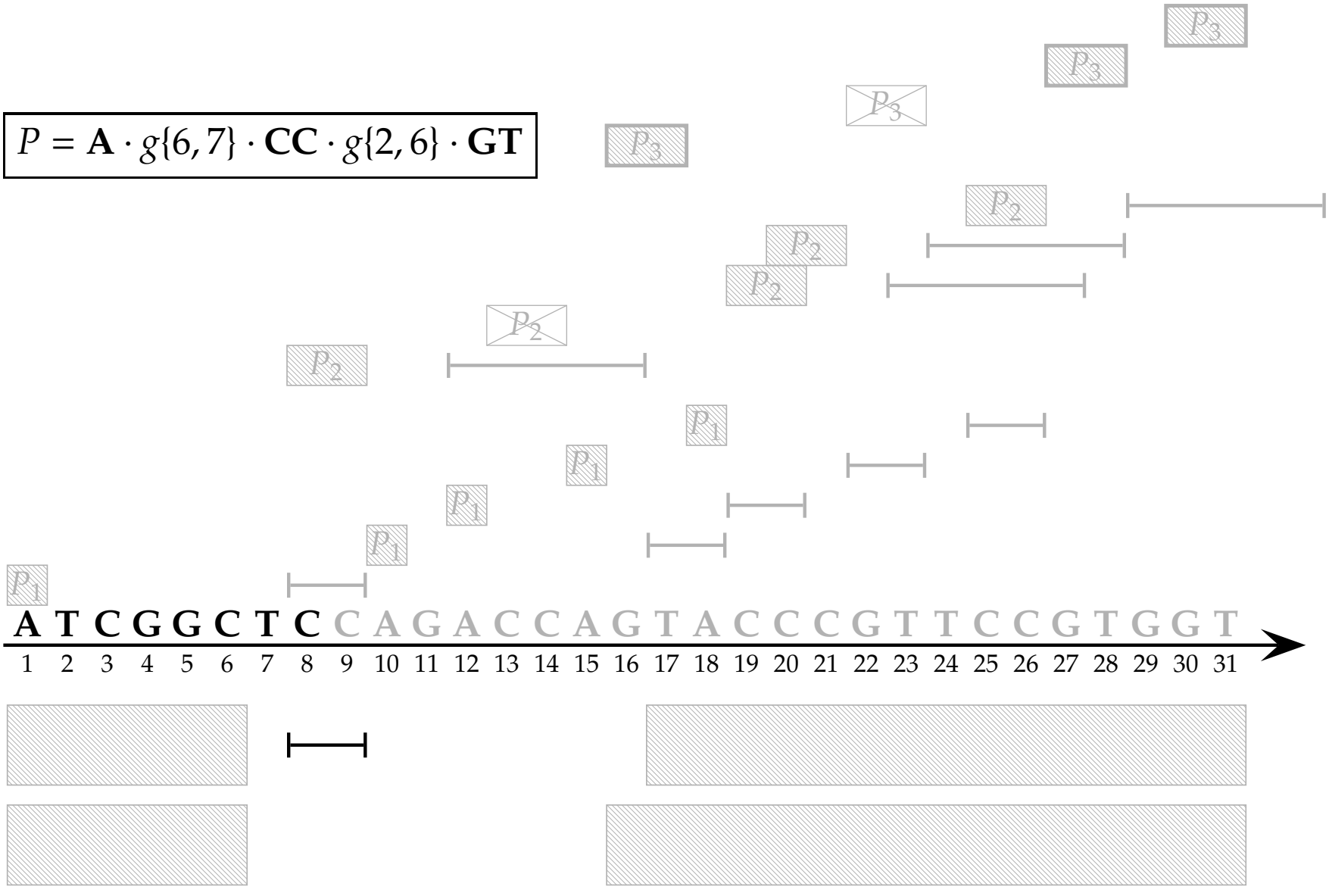

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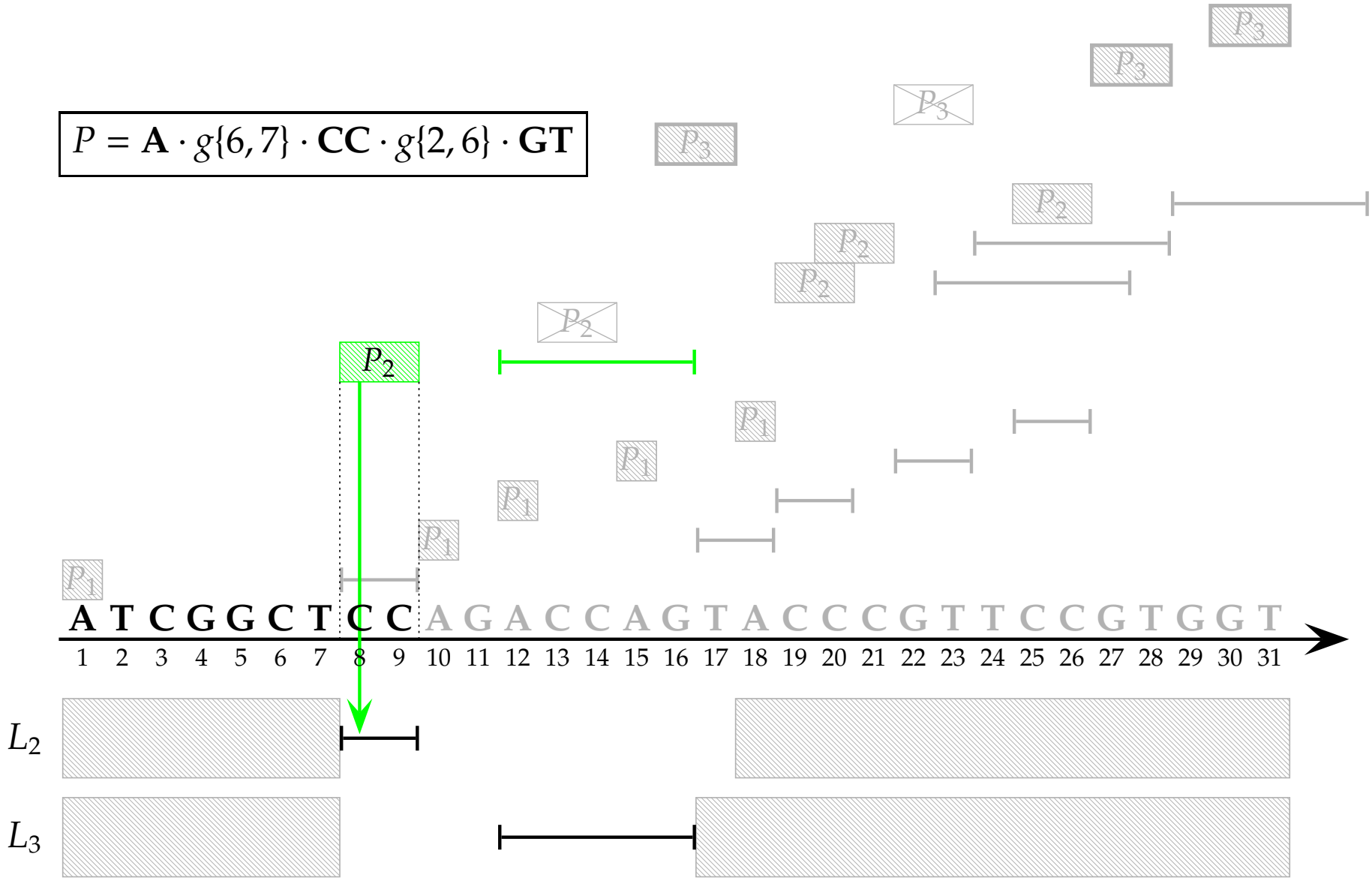

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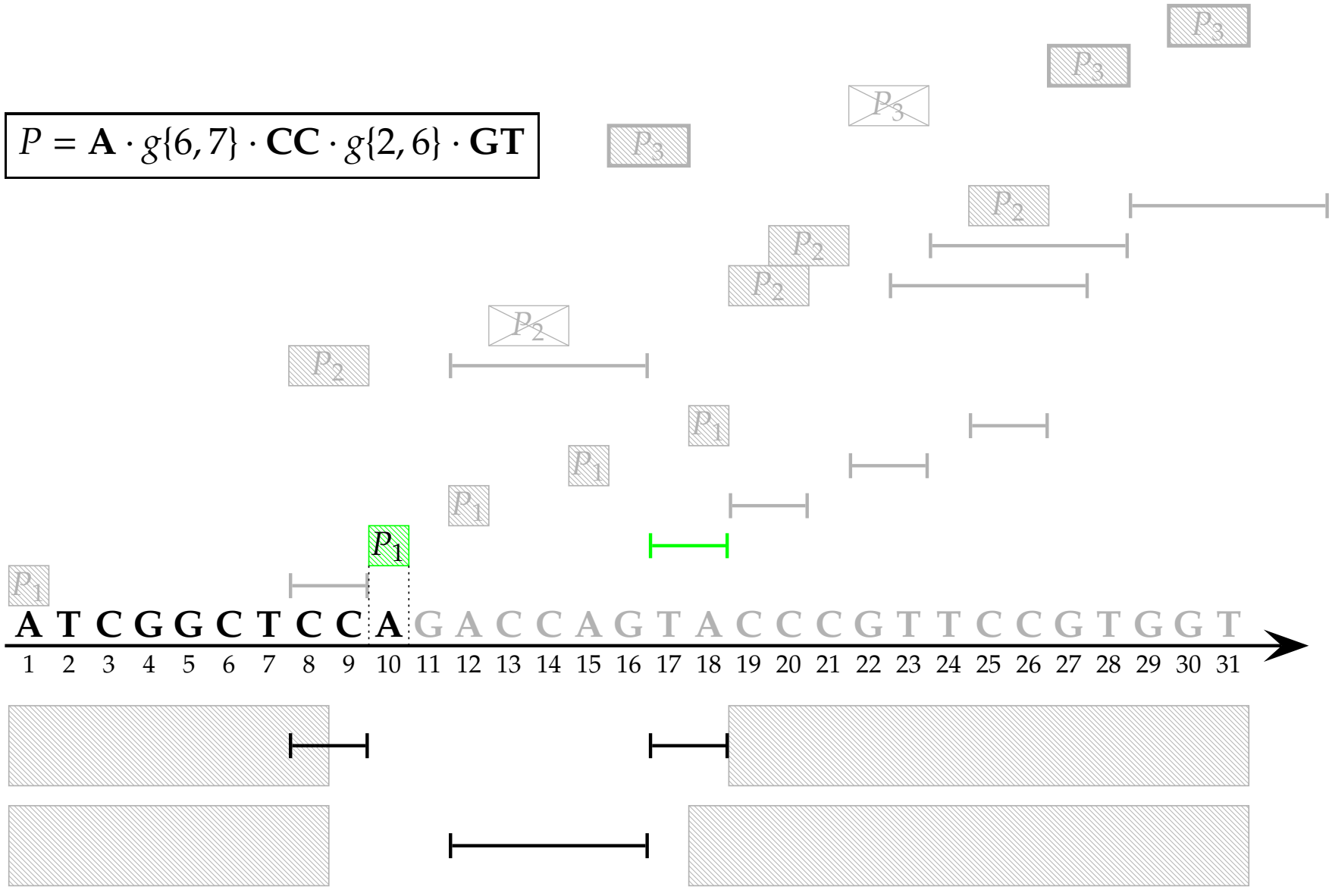

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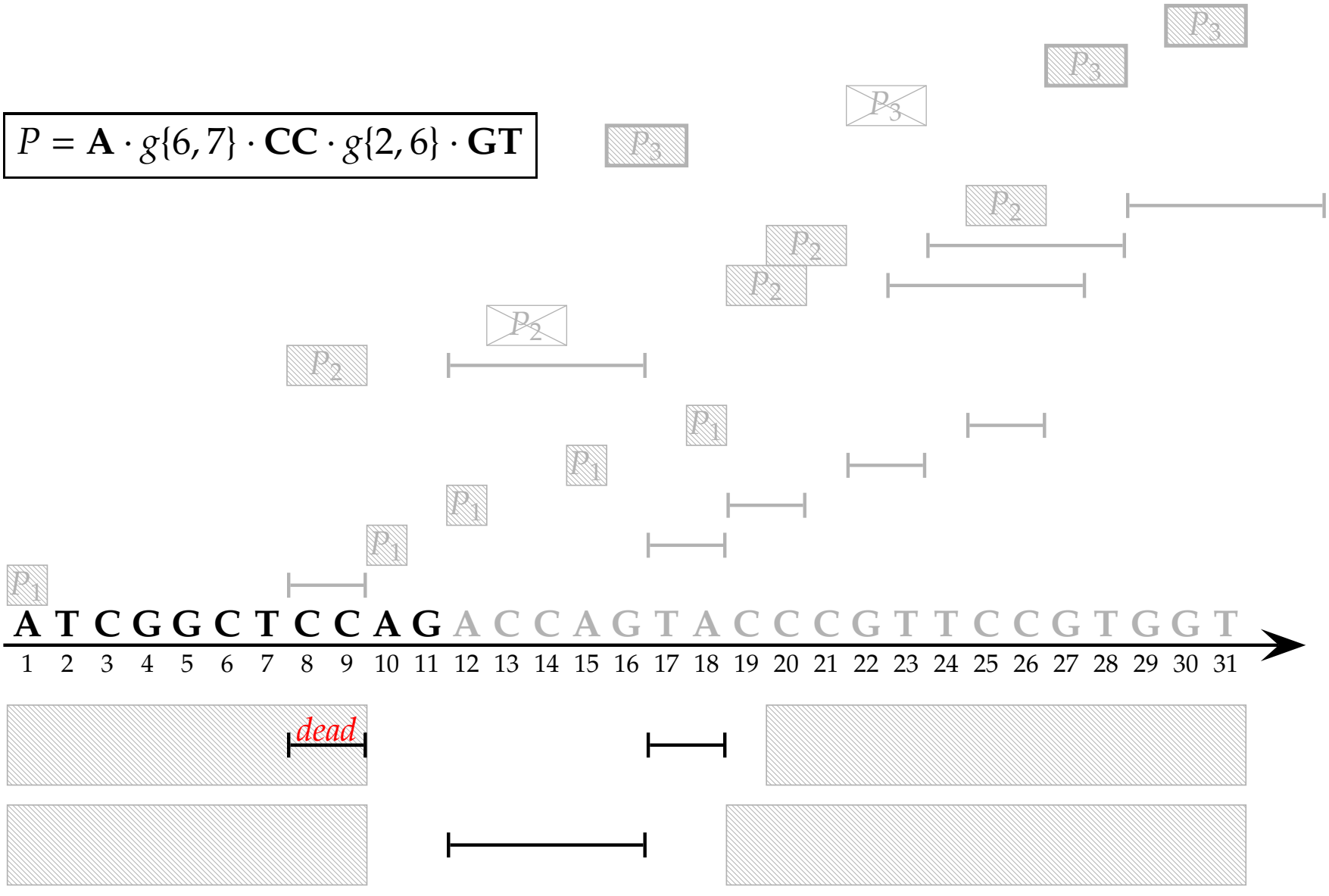

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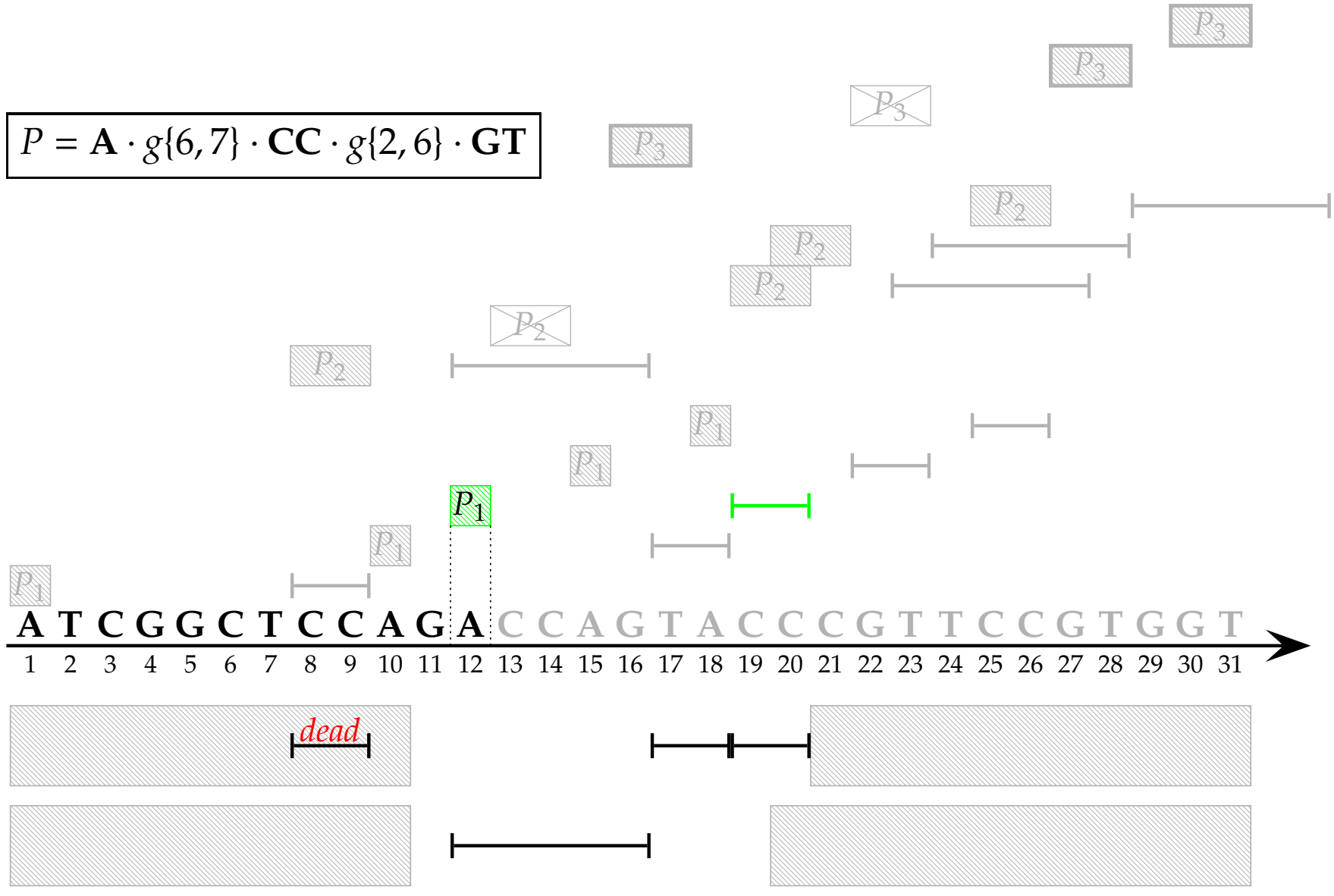

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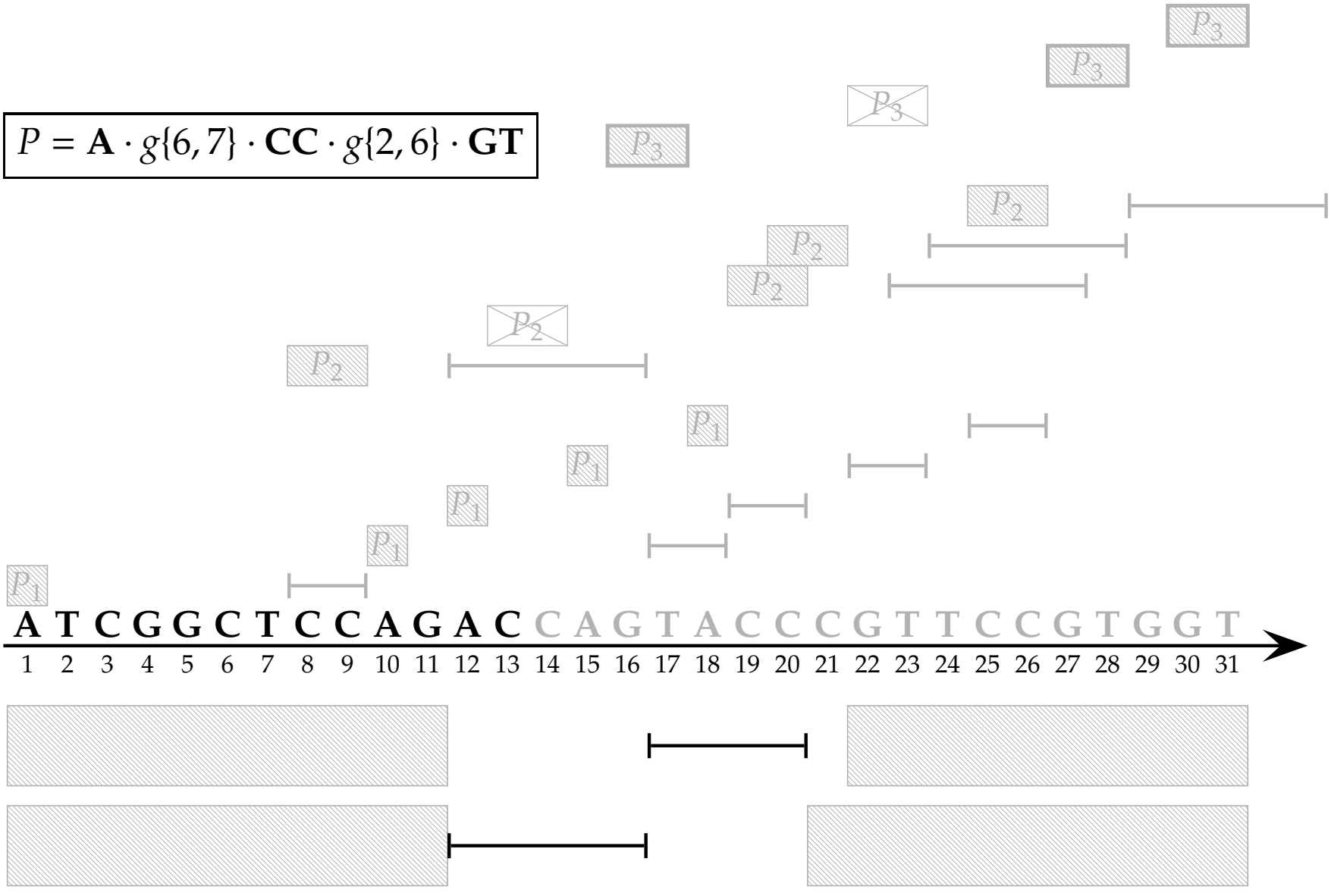

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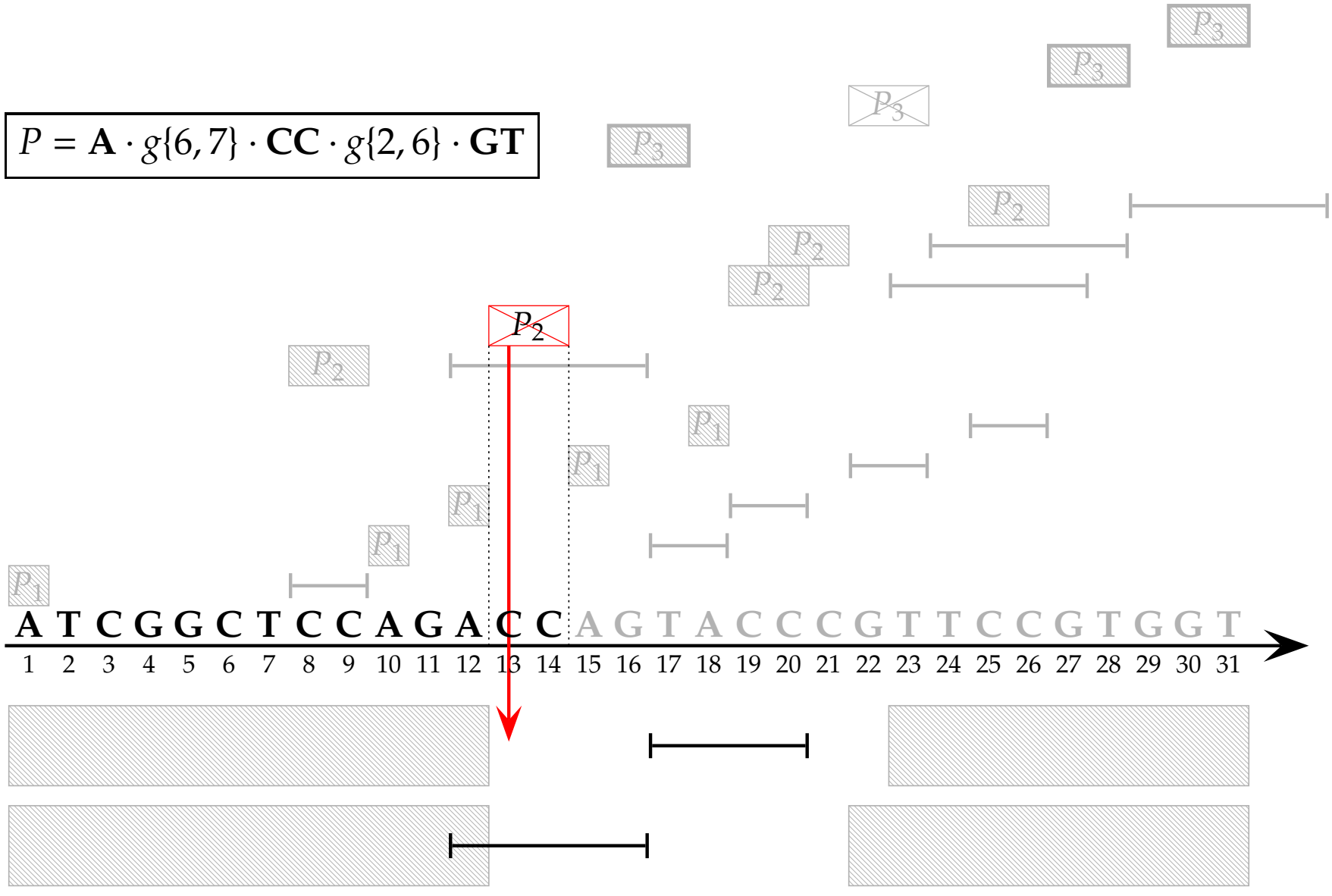

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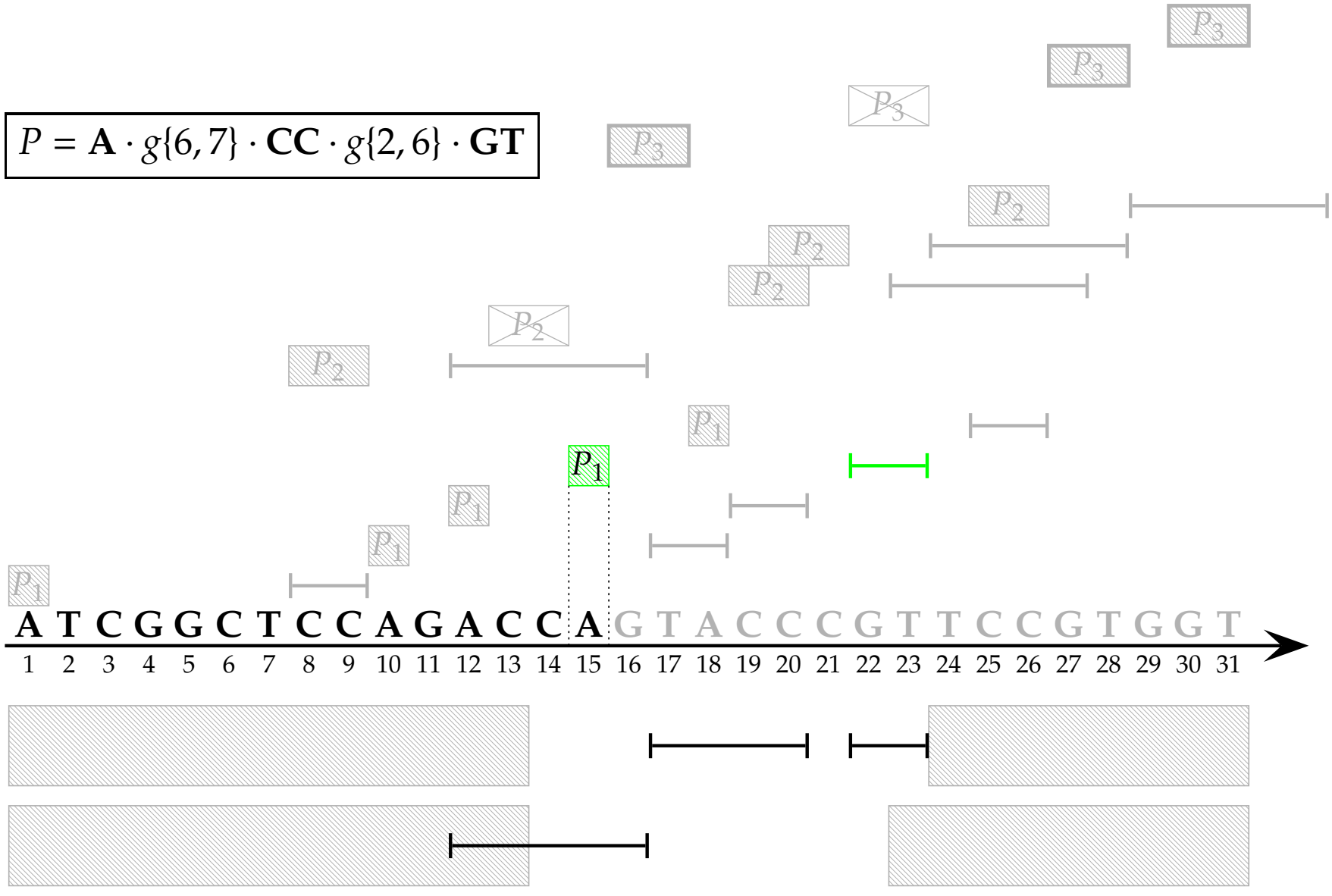

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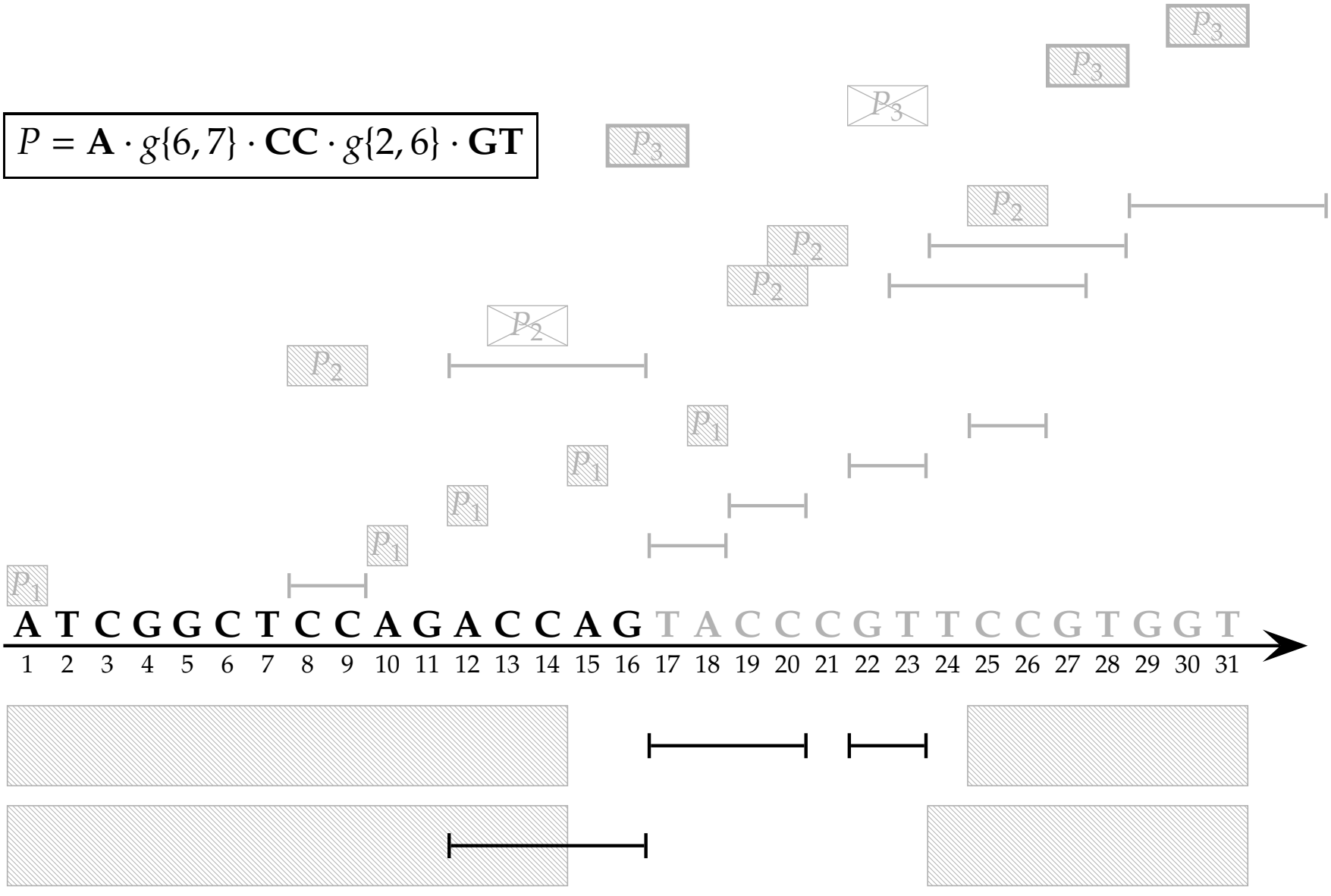

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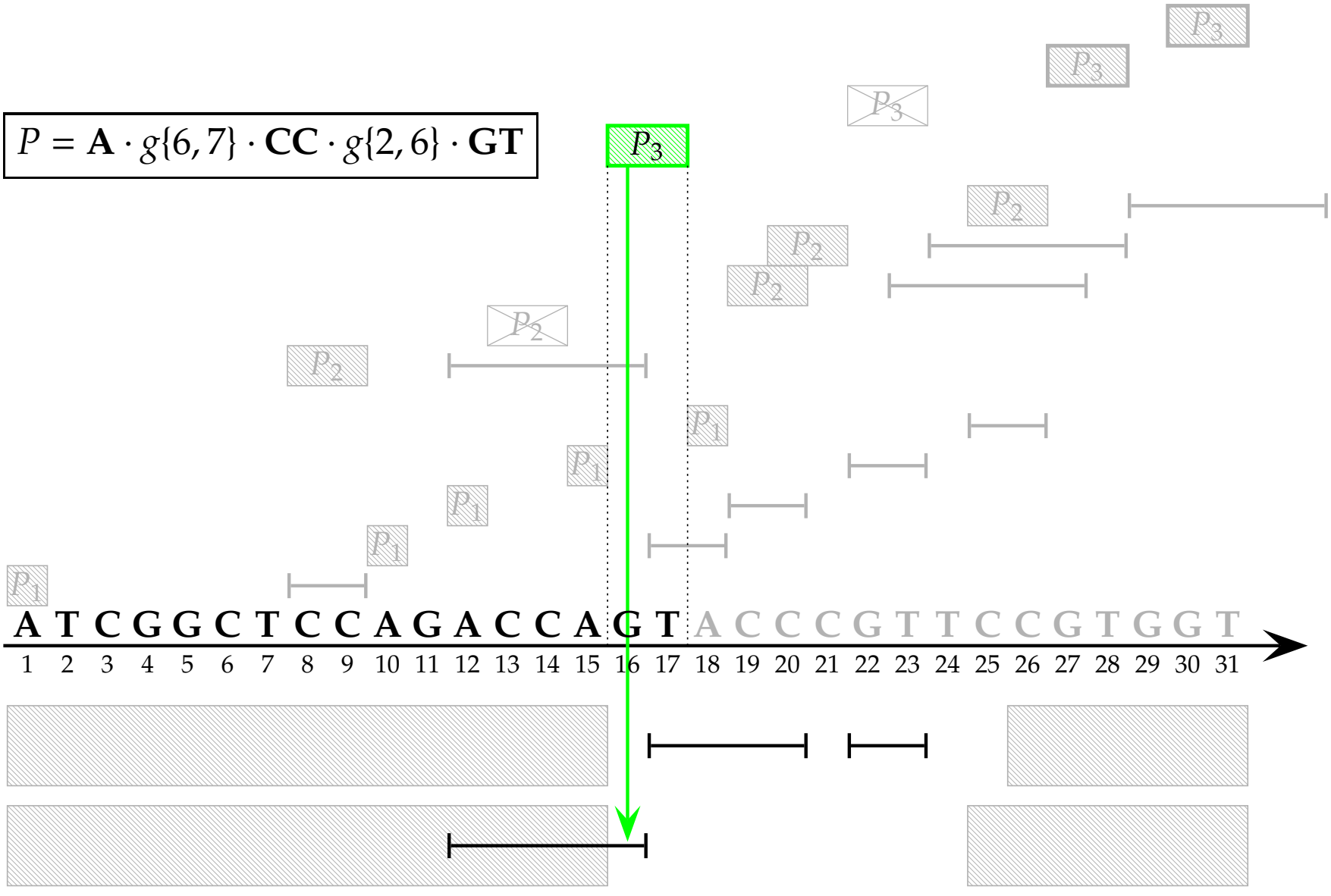

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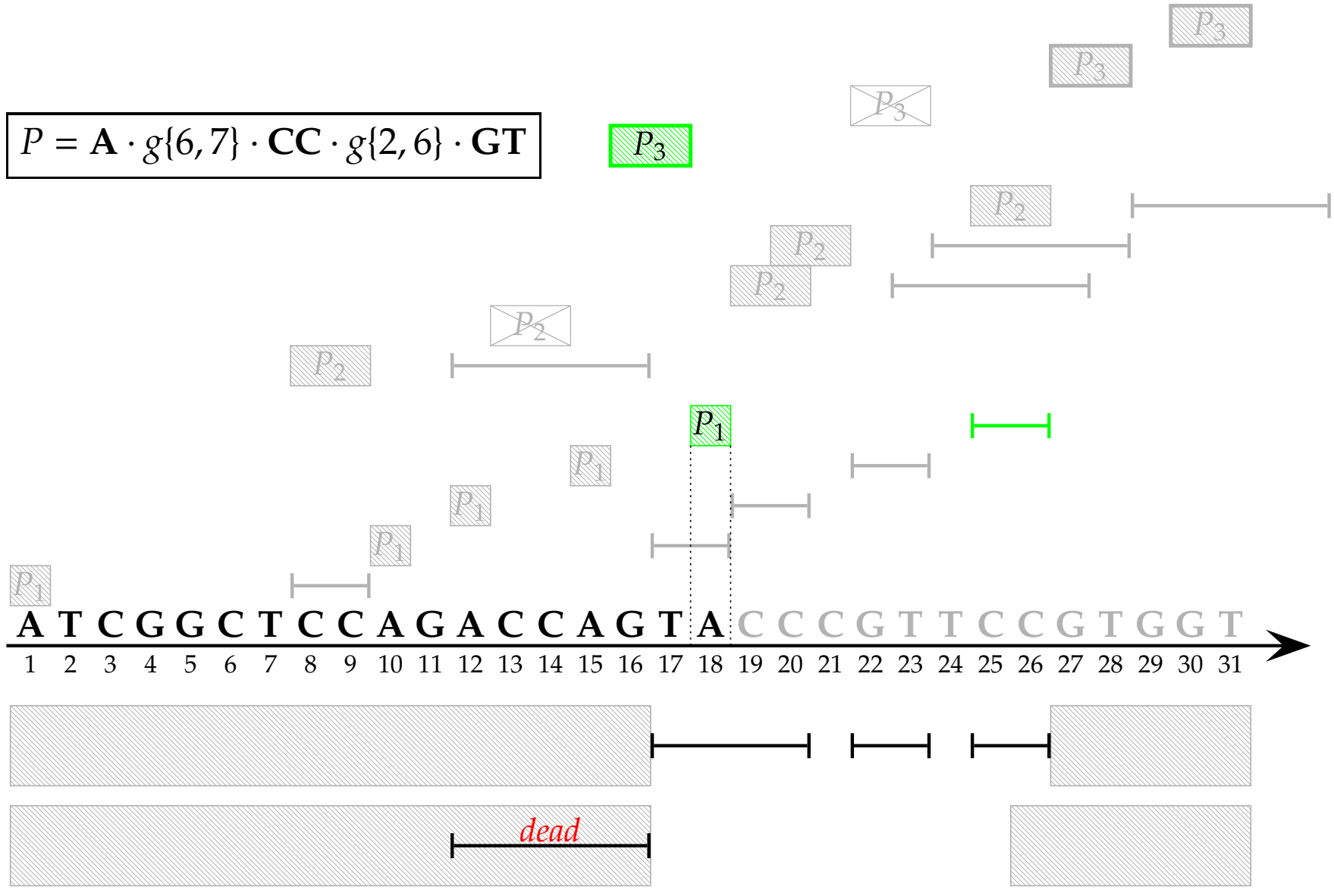

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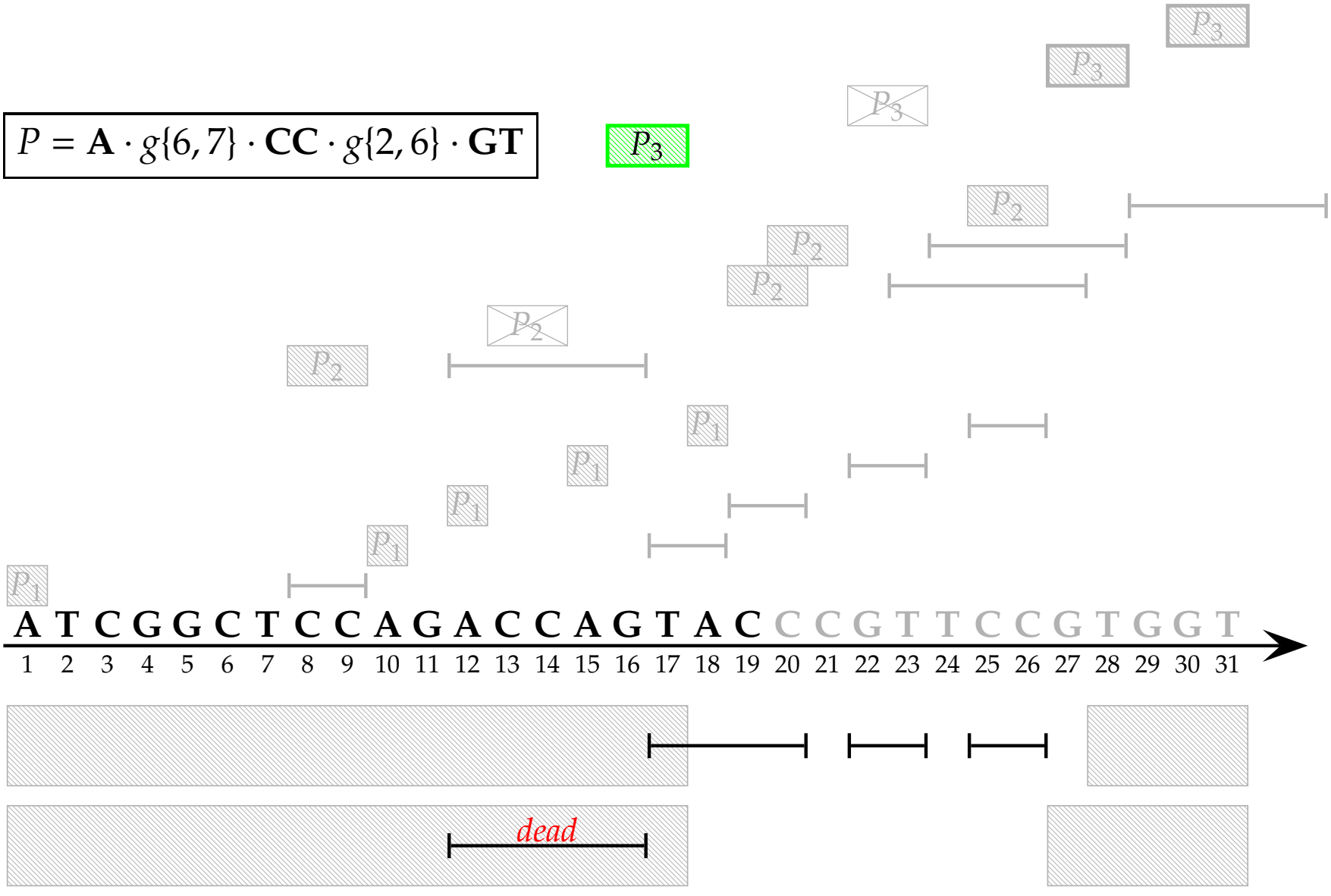

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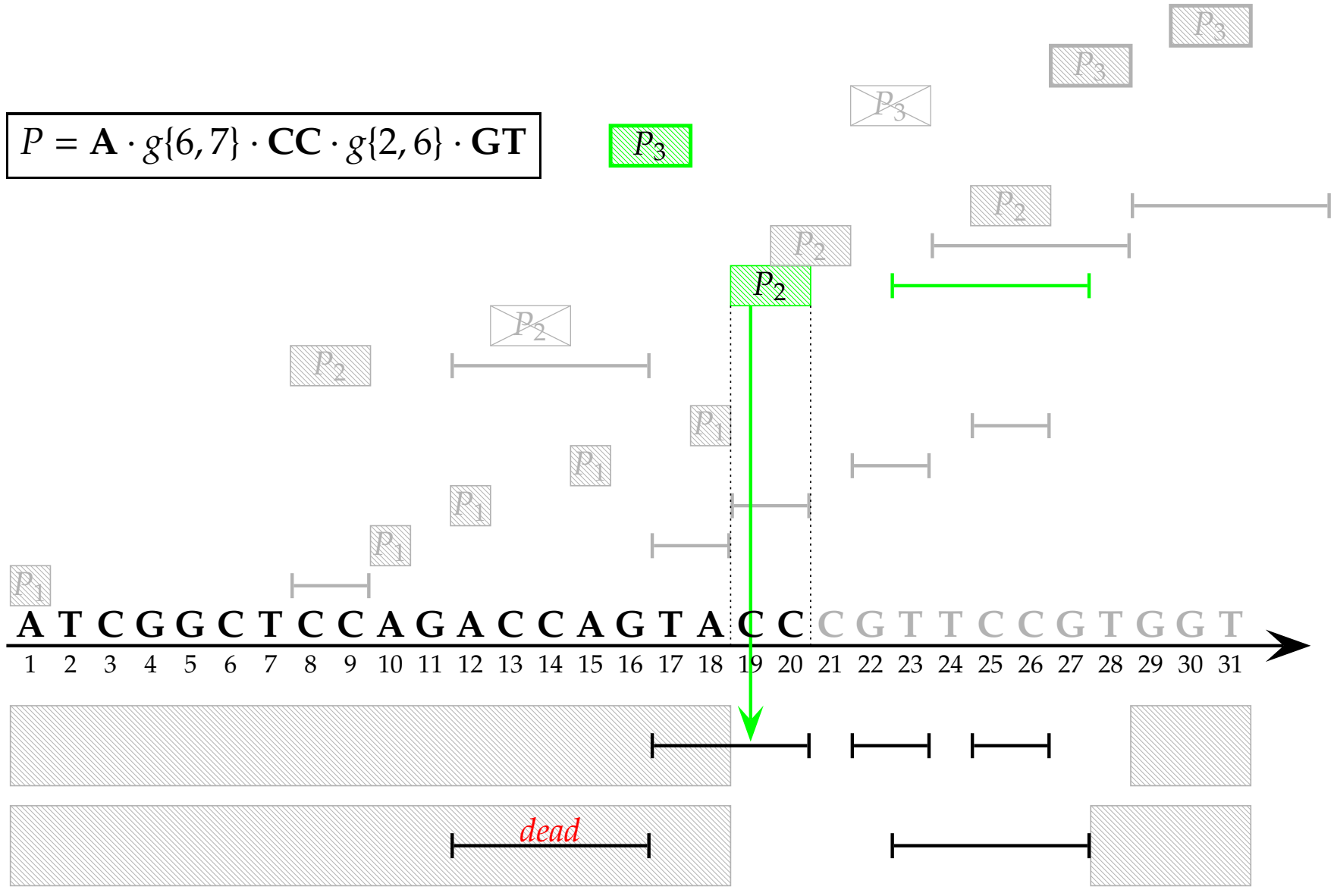

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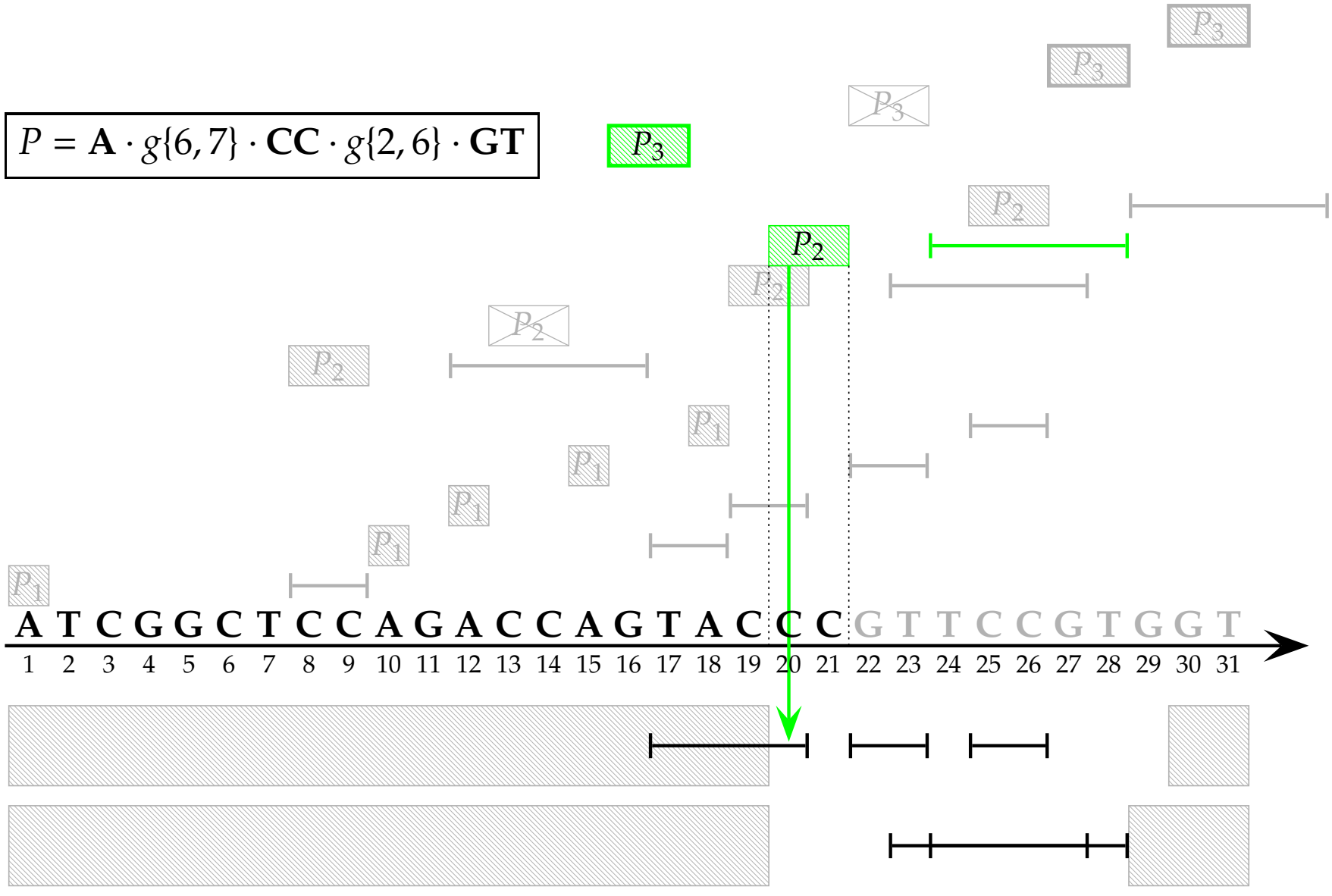

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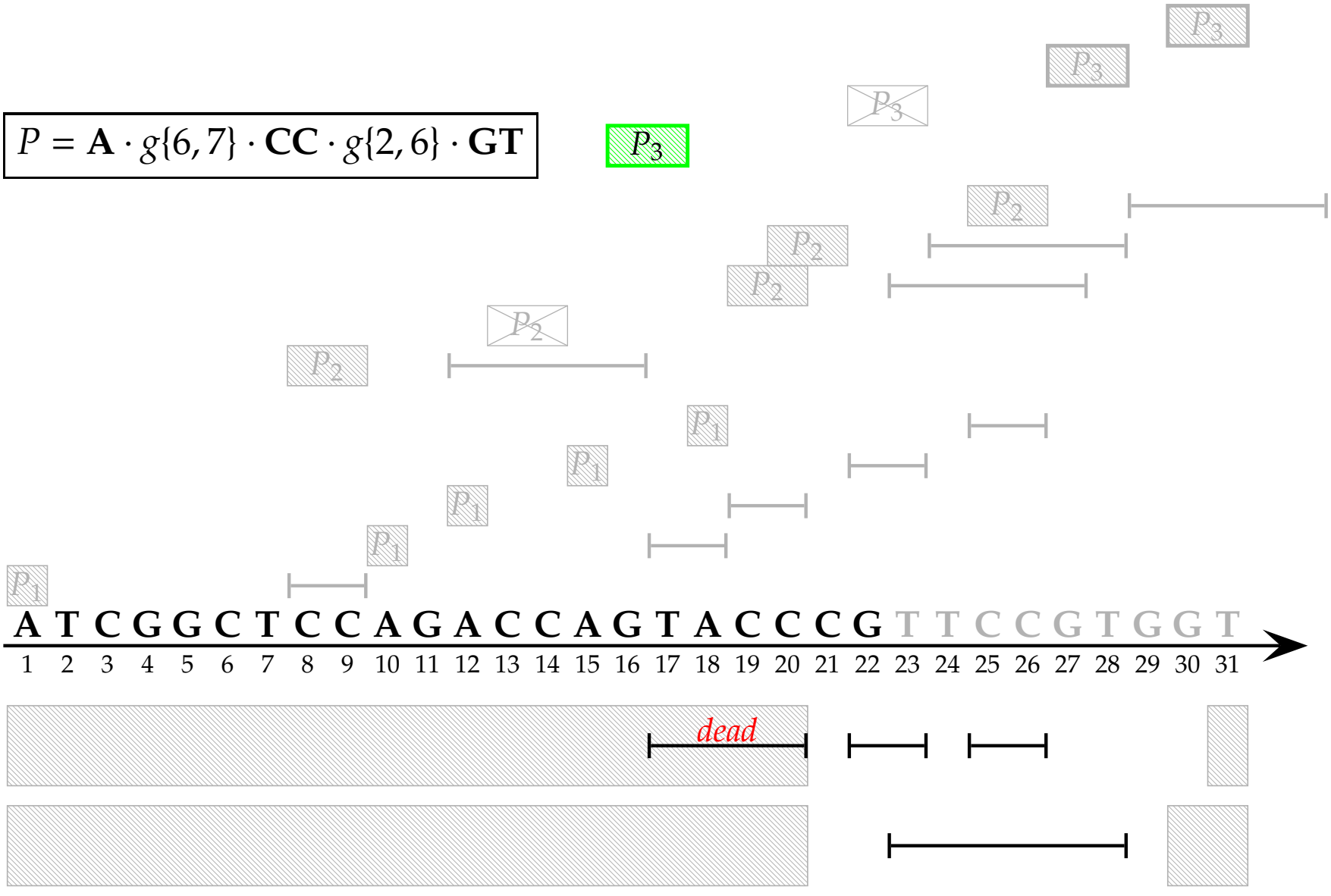

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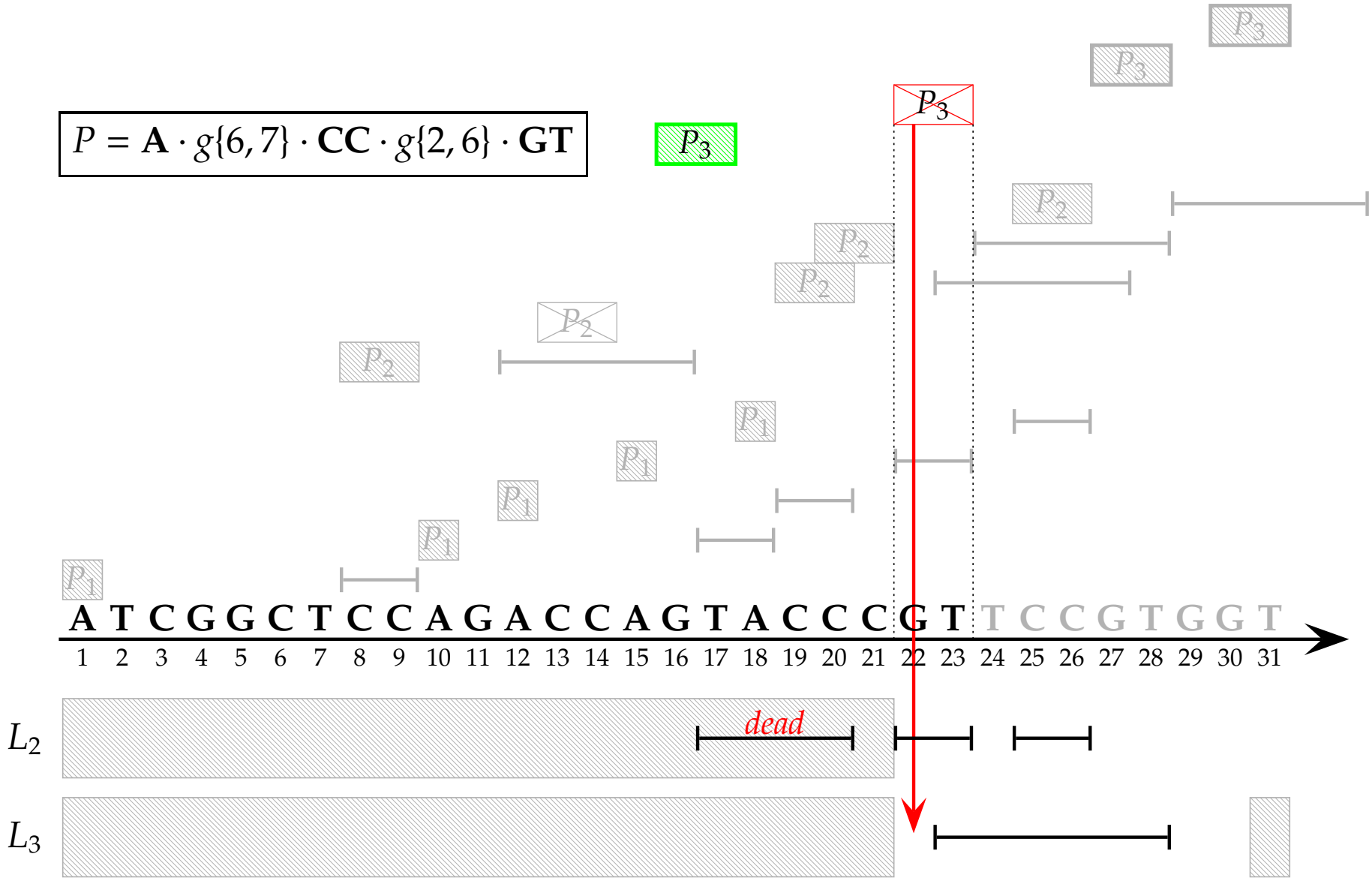

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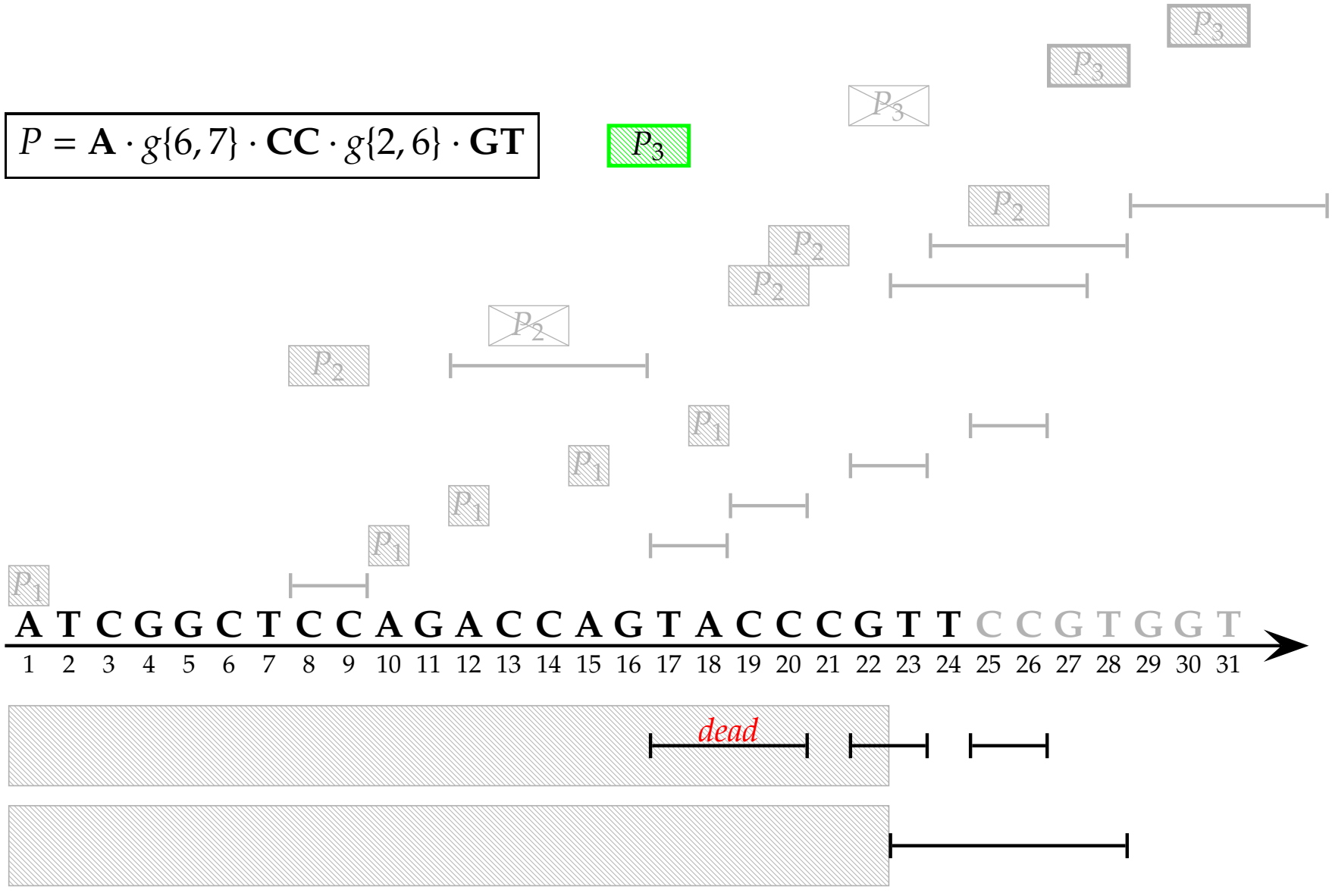

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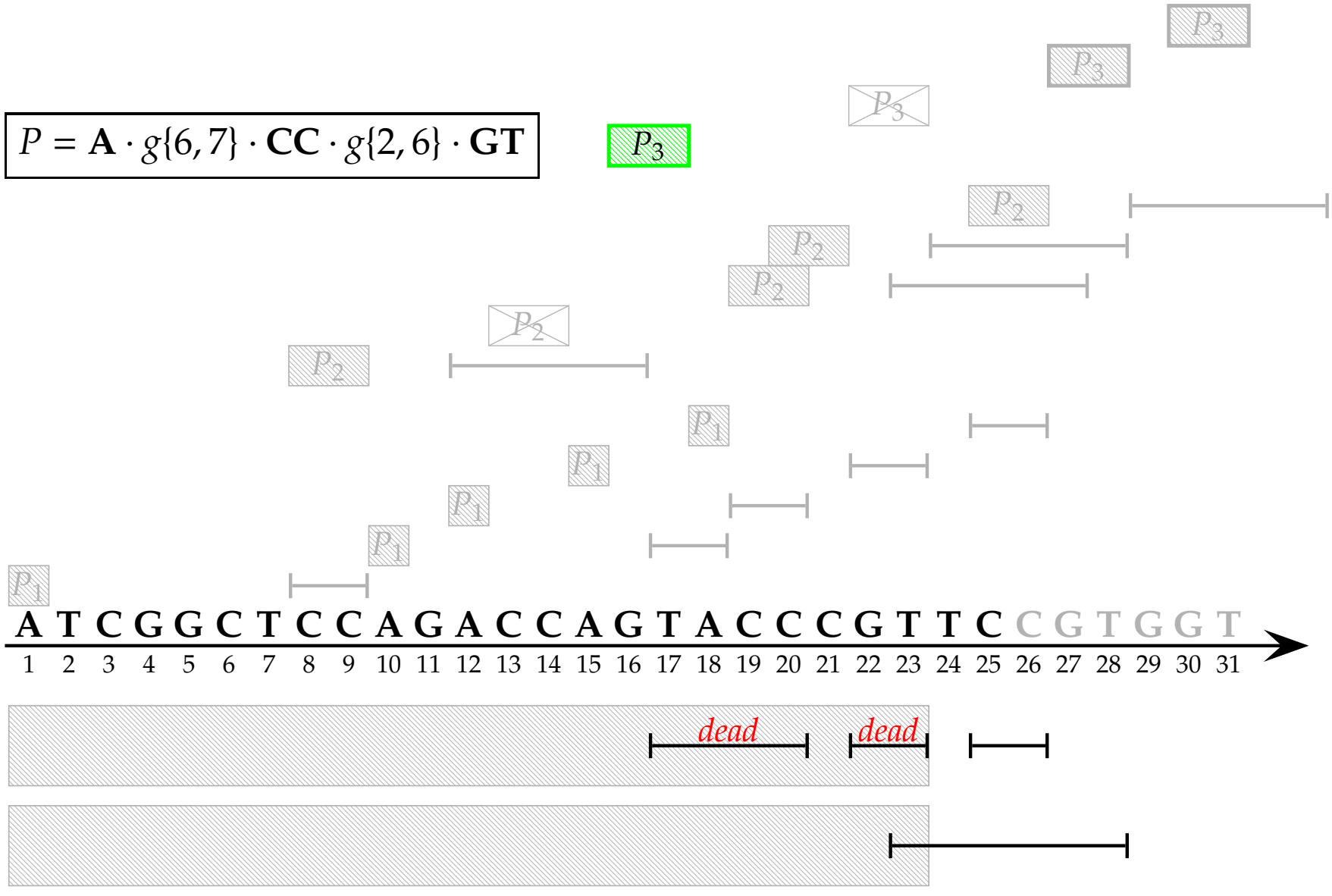

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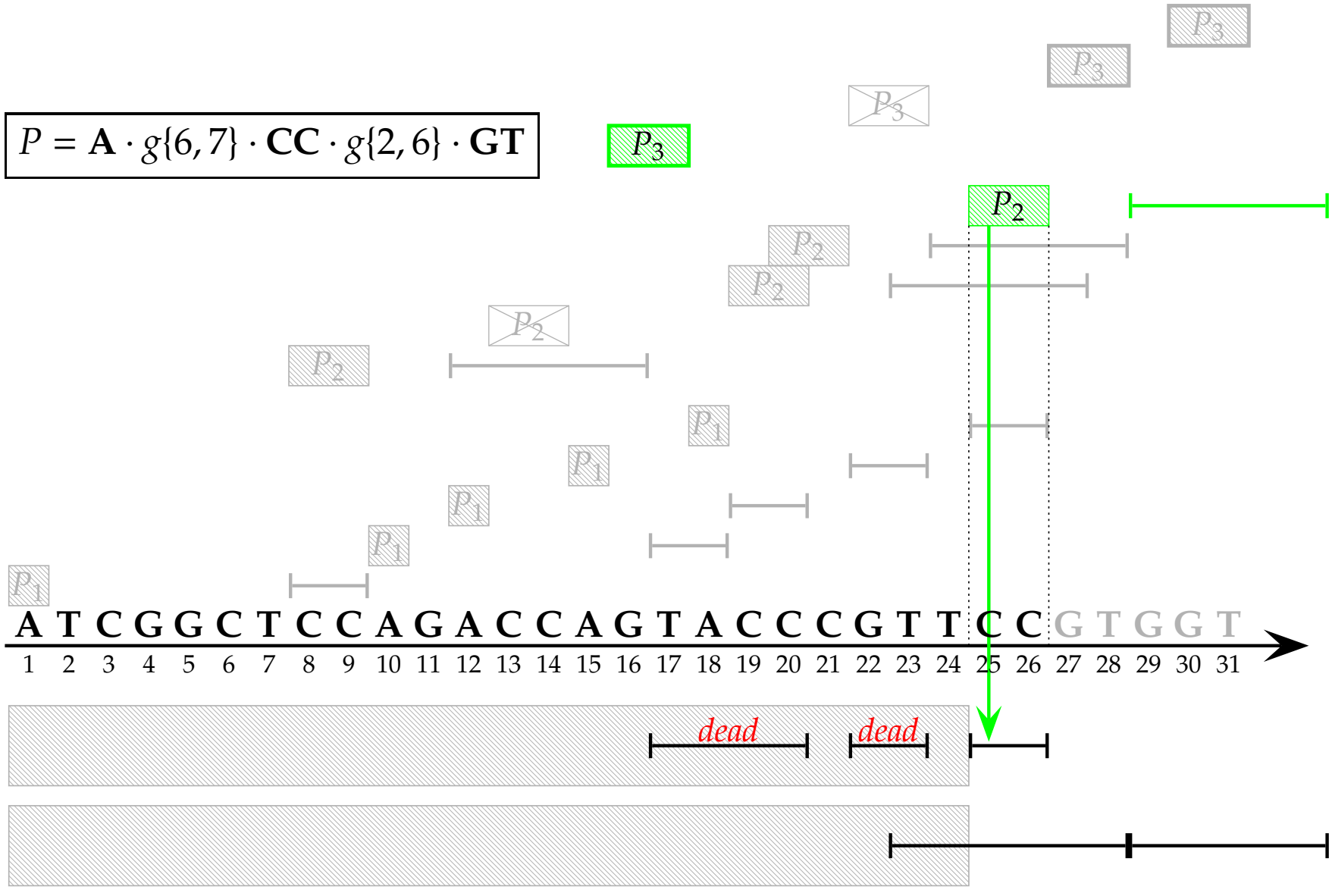

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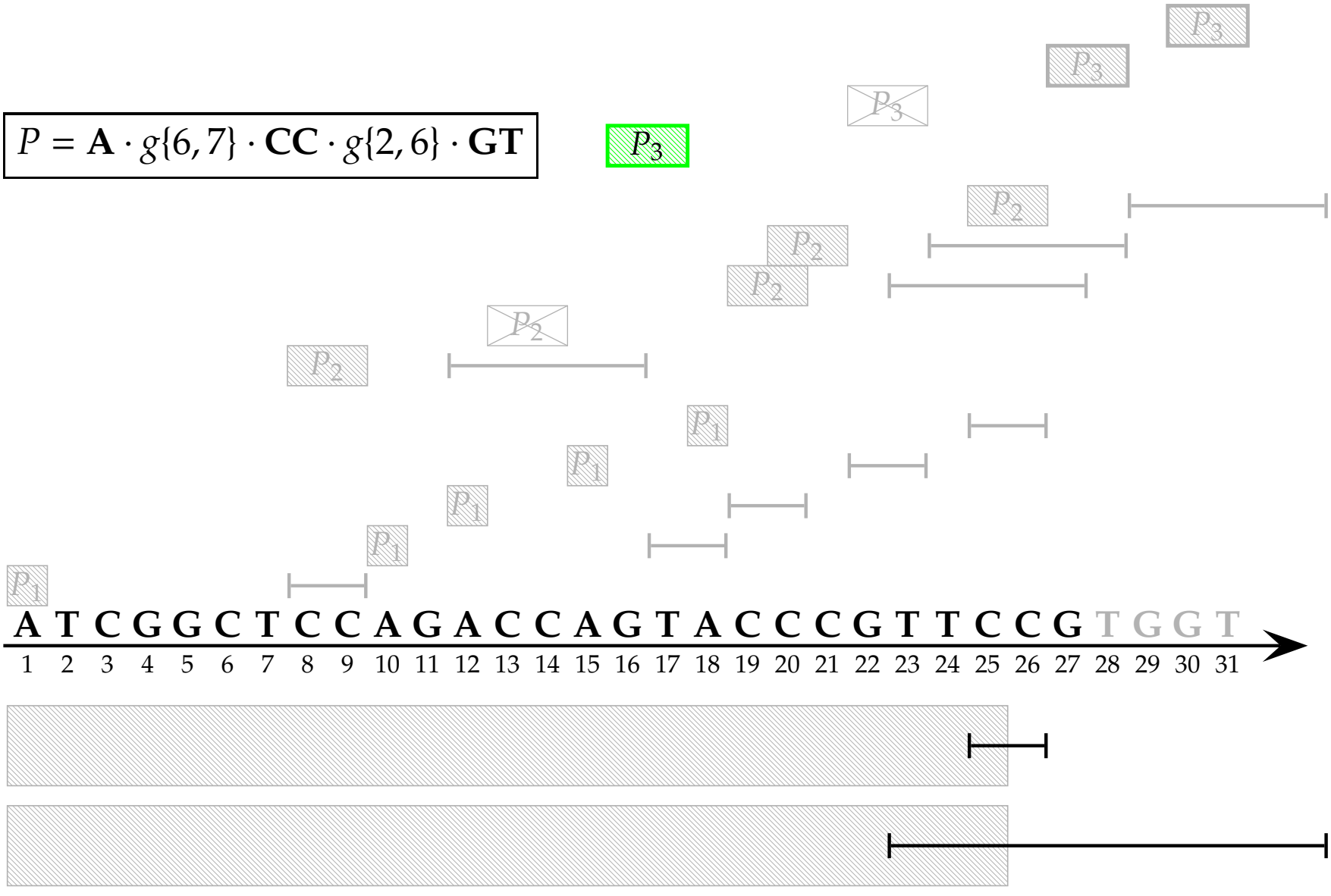

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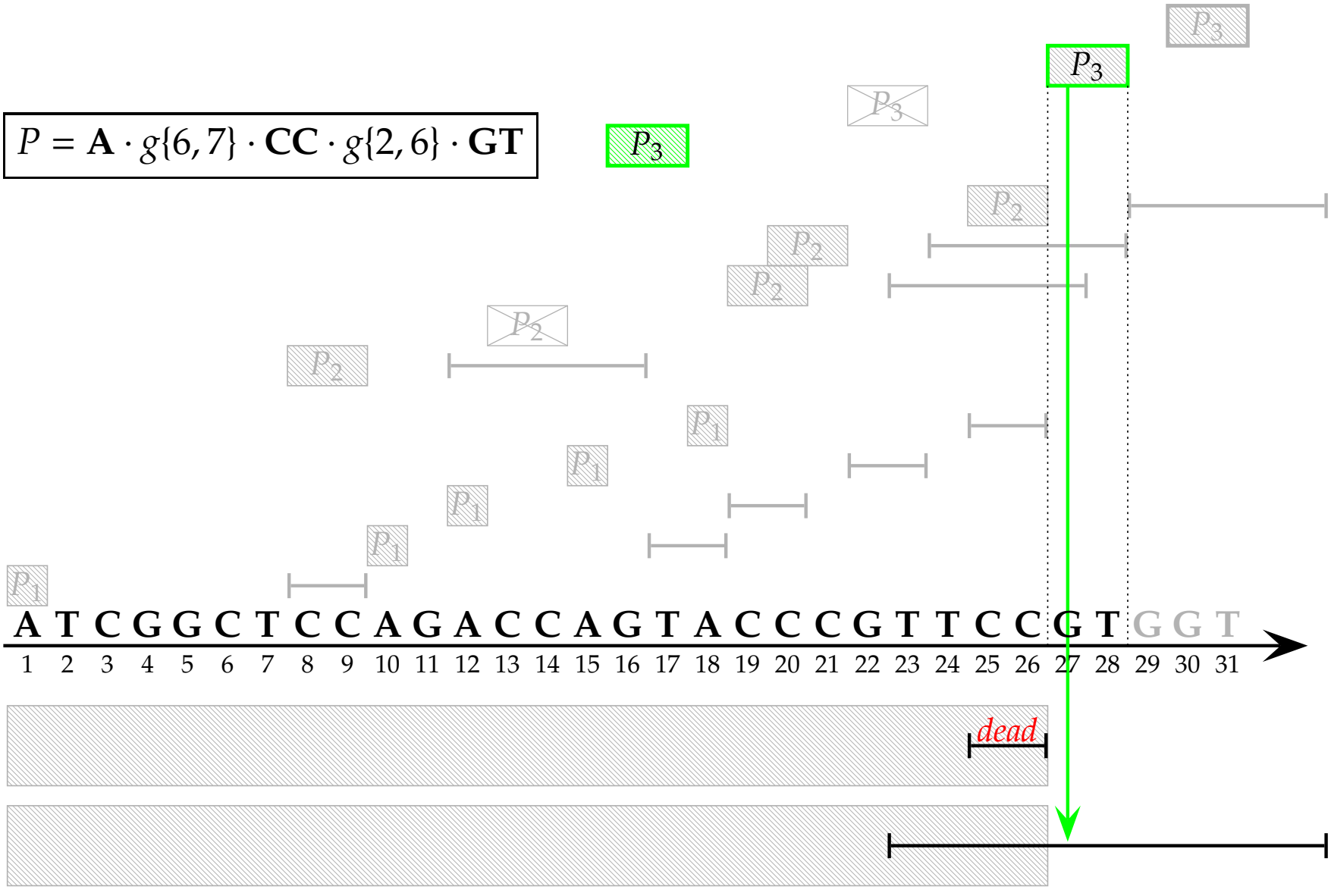

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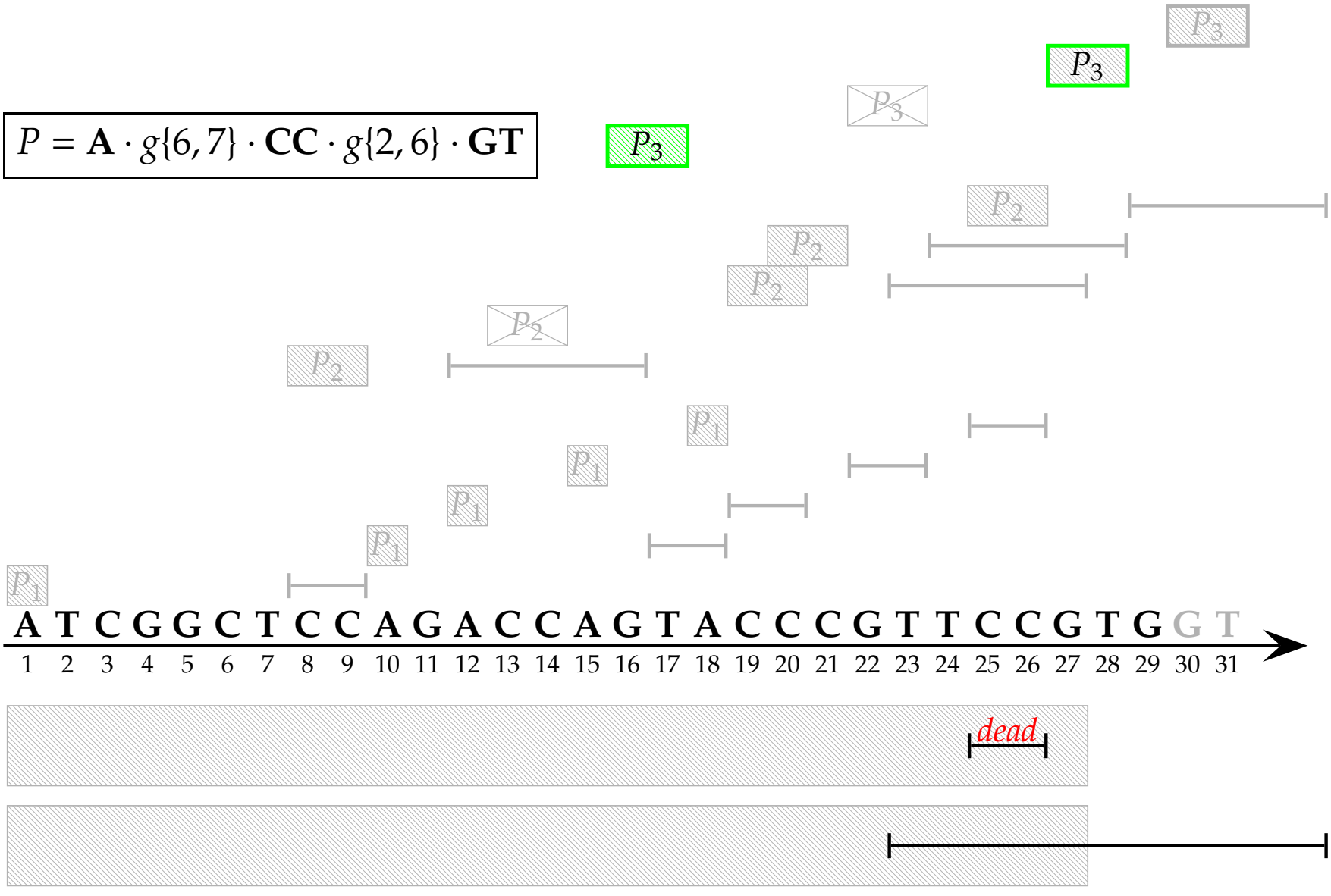

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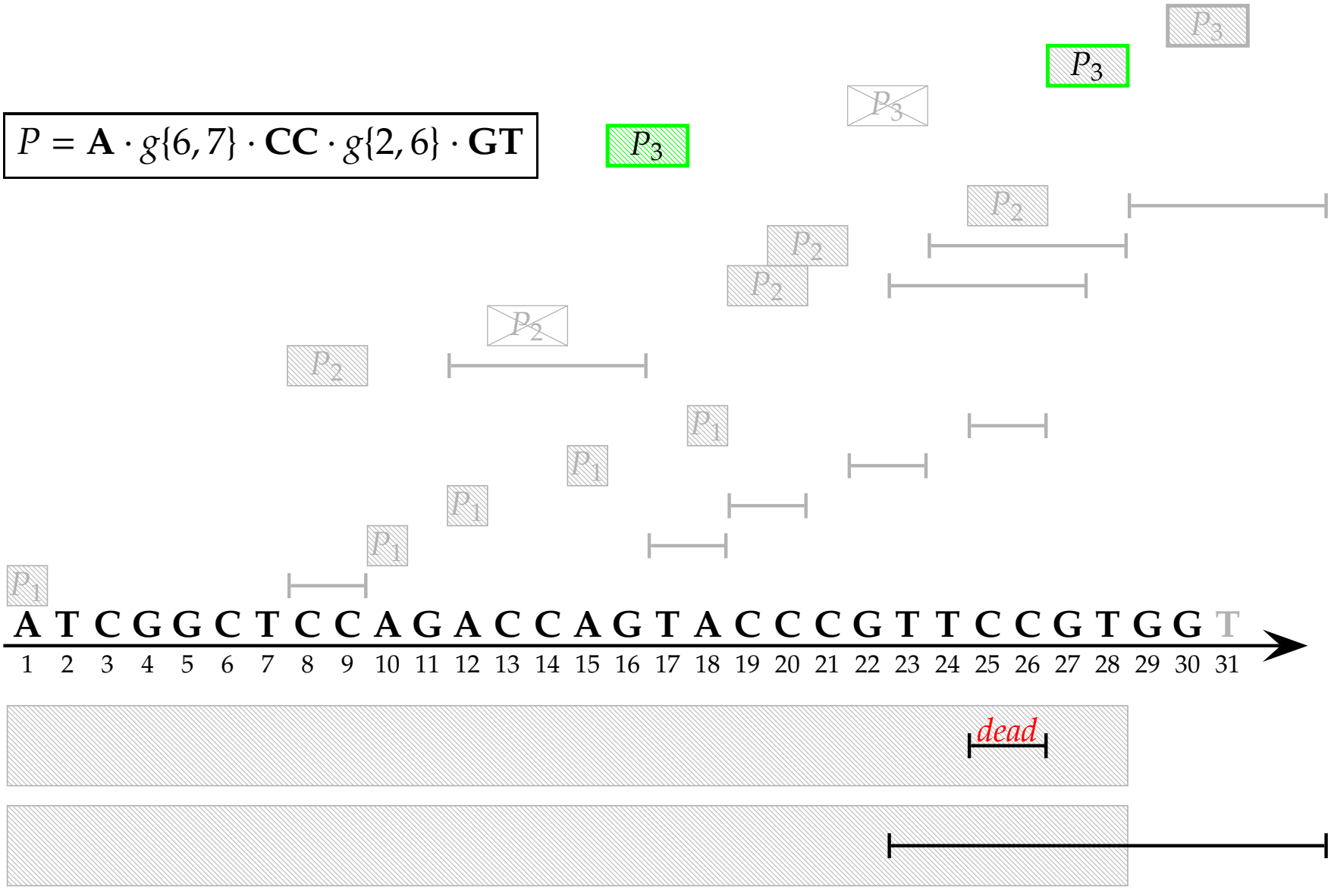

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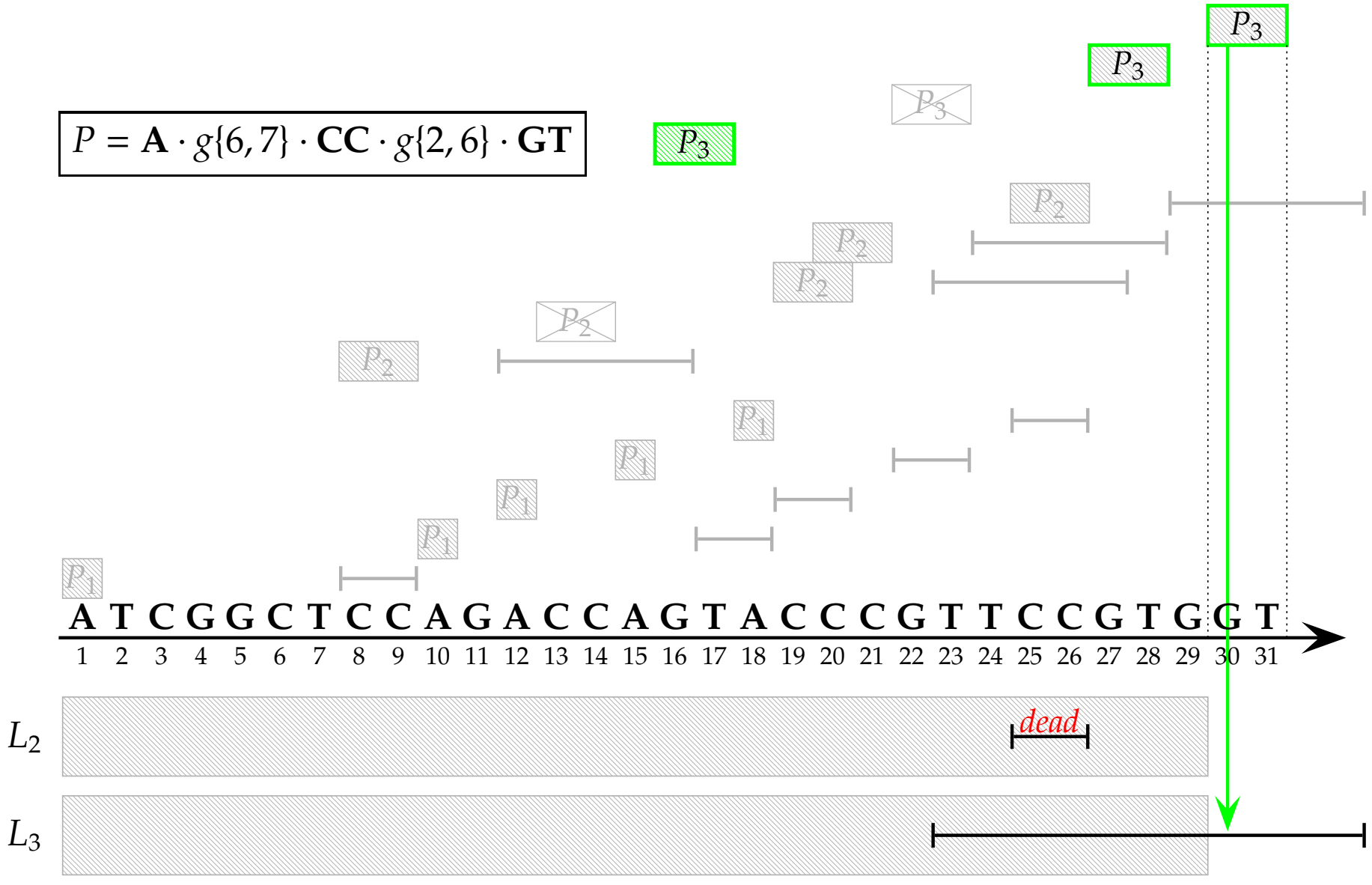

Illustrating the Algorithm

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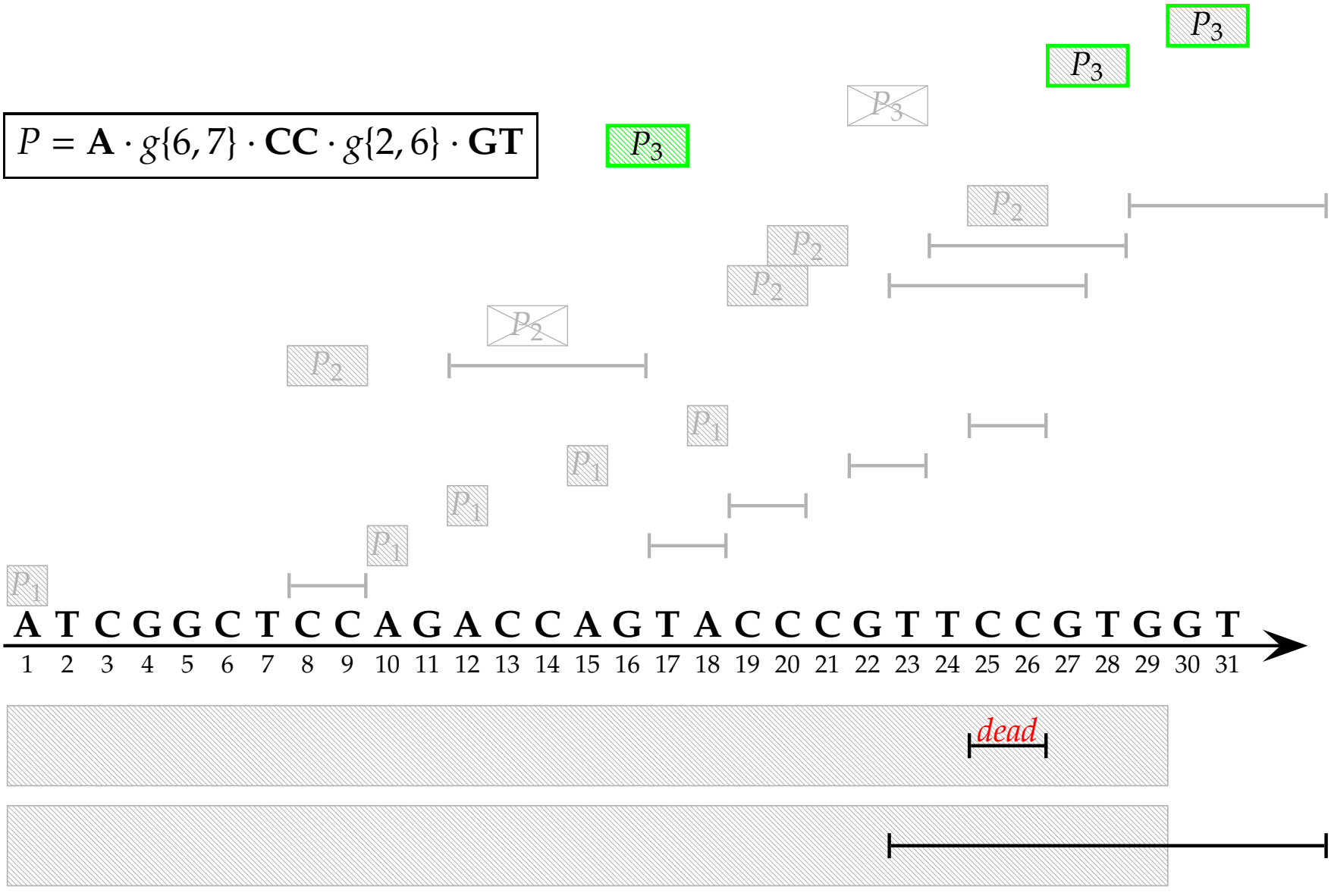

Time and Space

Claim: The algorithm runs in $O((n + m) \log k + \alpha)$ time and uses $O(m + A)$ space.

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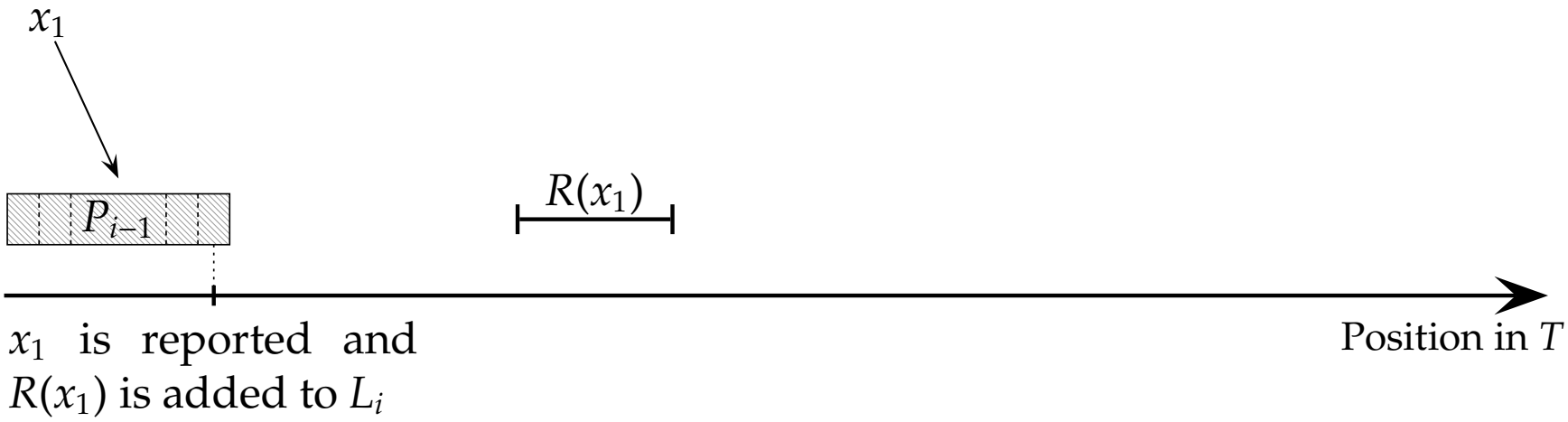
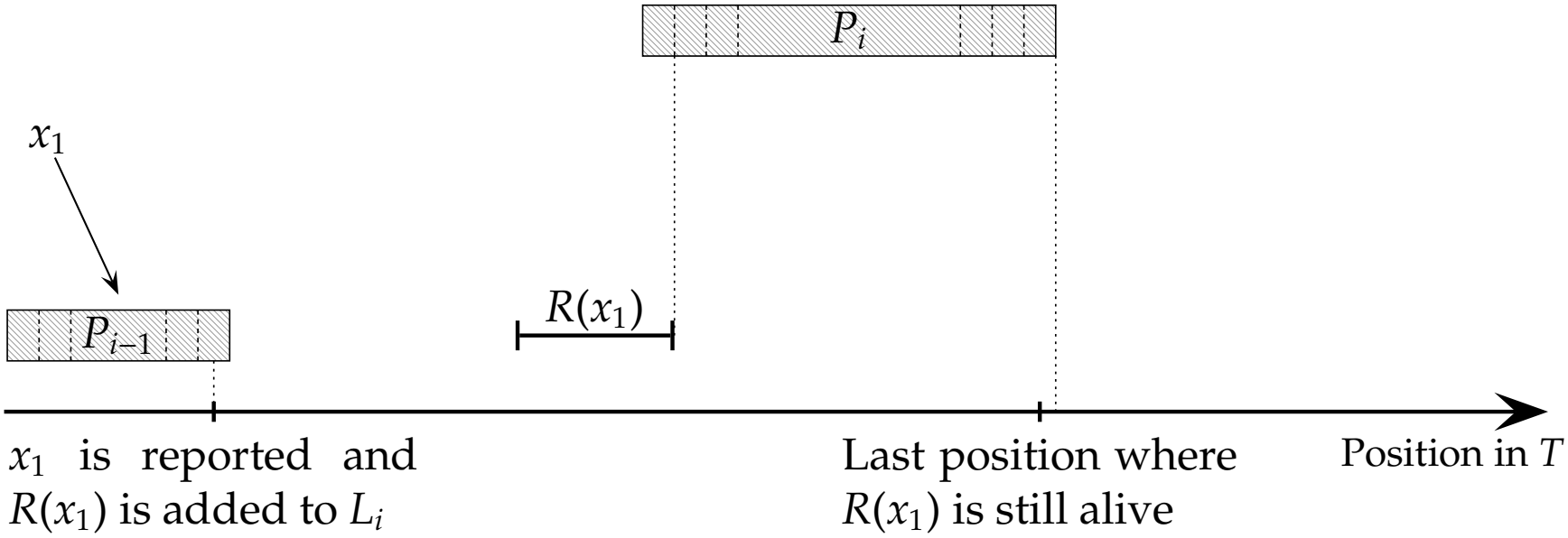
Space

- AC automaton takes $O(m)$ space.
- How much space is used by $L_2, \ldots, L_k$?

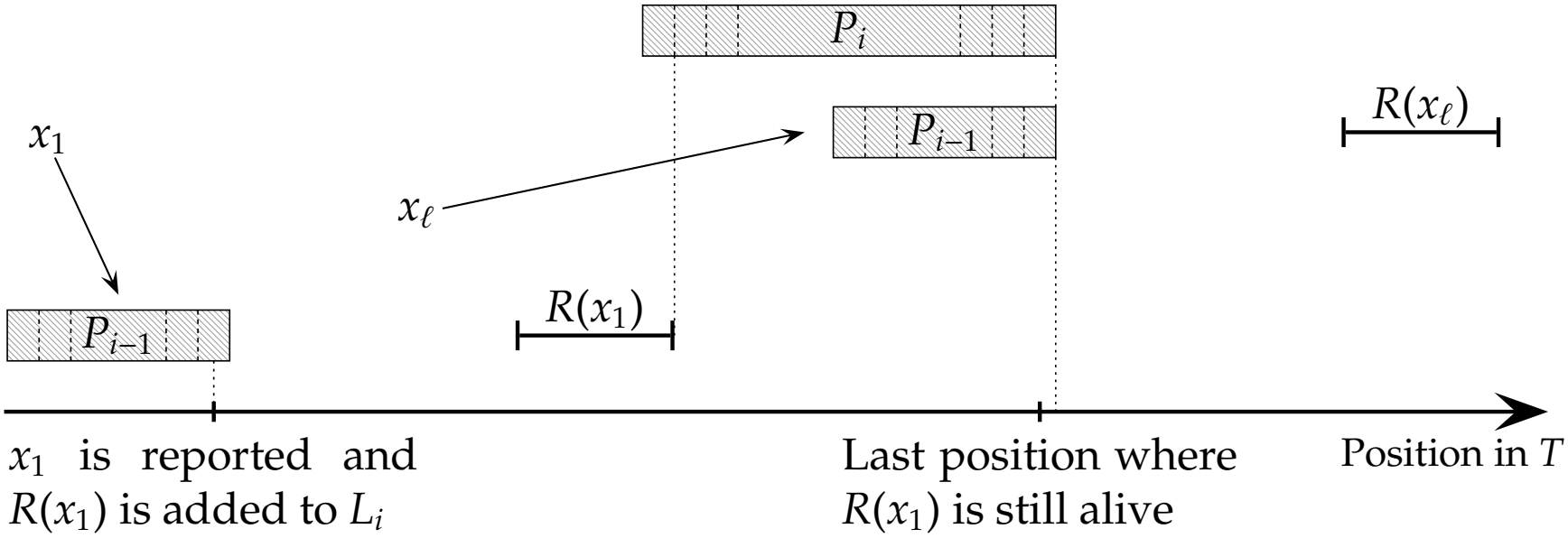
Maximum Size of L_i



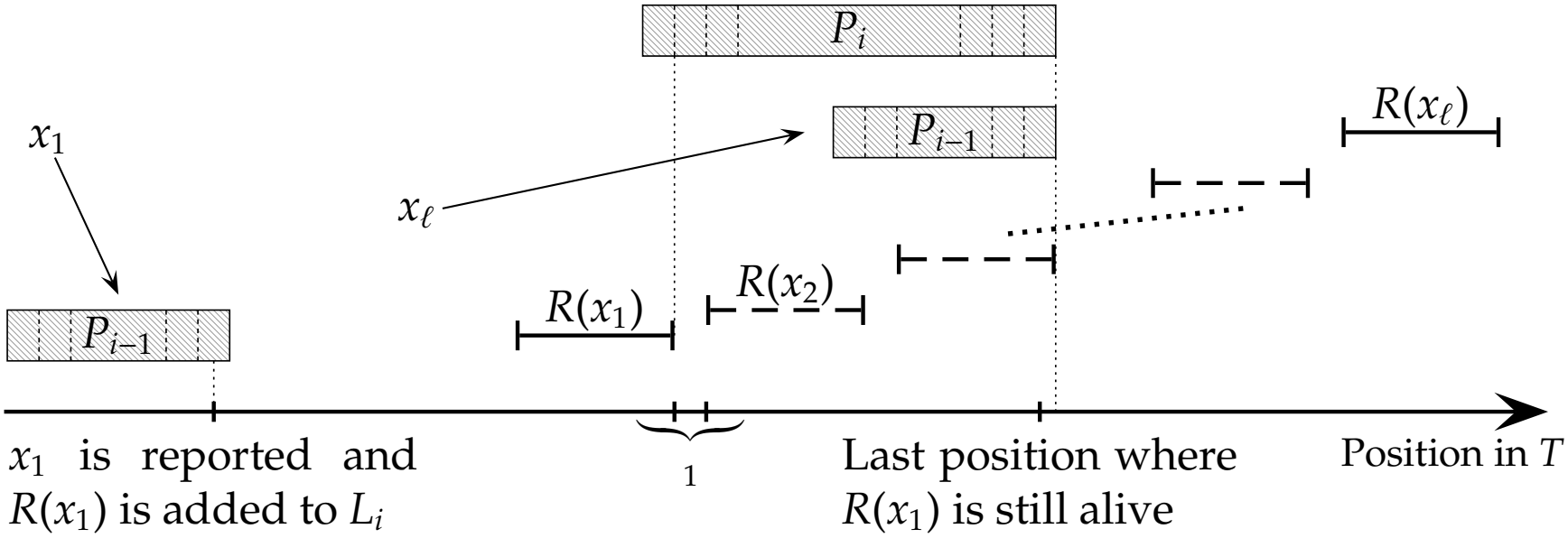
Maximum Size of L_i



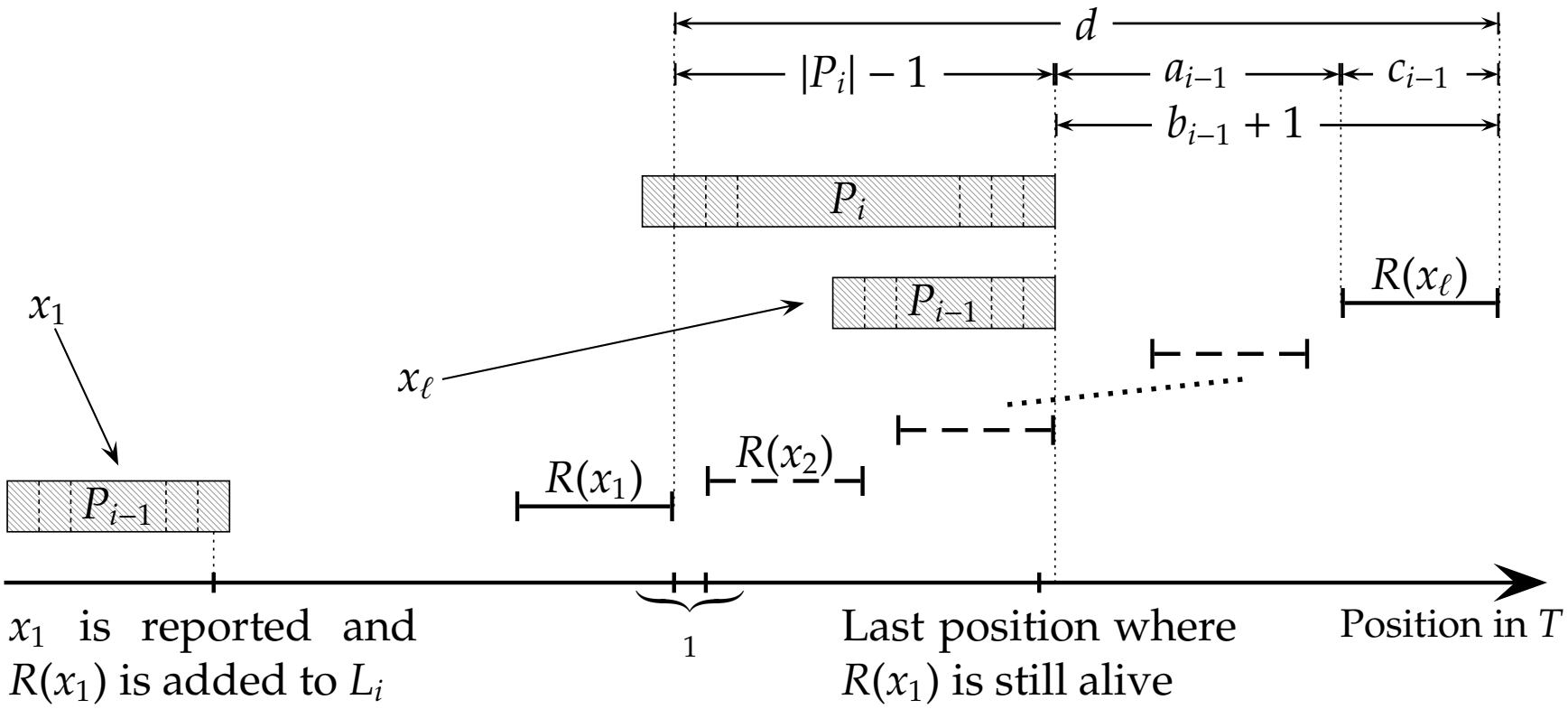
Maximum Size of L_i



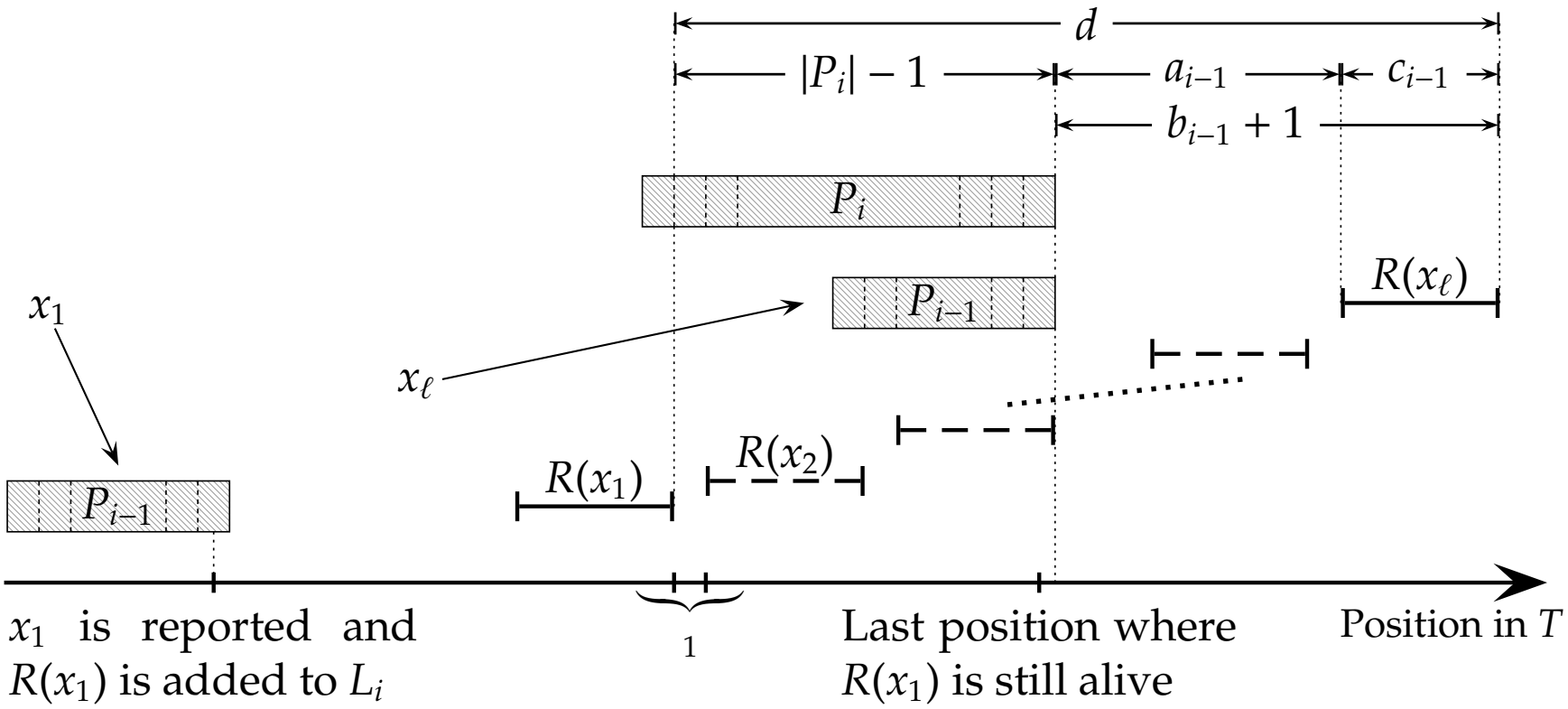
Maximum Size of L_i



Maximum Size of L_i



Maximum Size of L_i



$$|L_i| \leq \left\lfloor \frac{d}{c_{i-1}+1} \right\rfloor + 1 = \left\lfloor \frac{2c_{i-1} + |P_i| + a_{i-1}}{c_{i-1}+1} \right\rfloor = O(|P_i| + a_{i-1}) \; .$$

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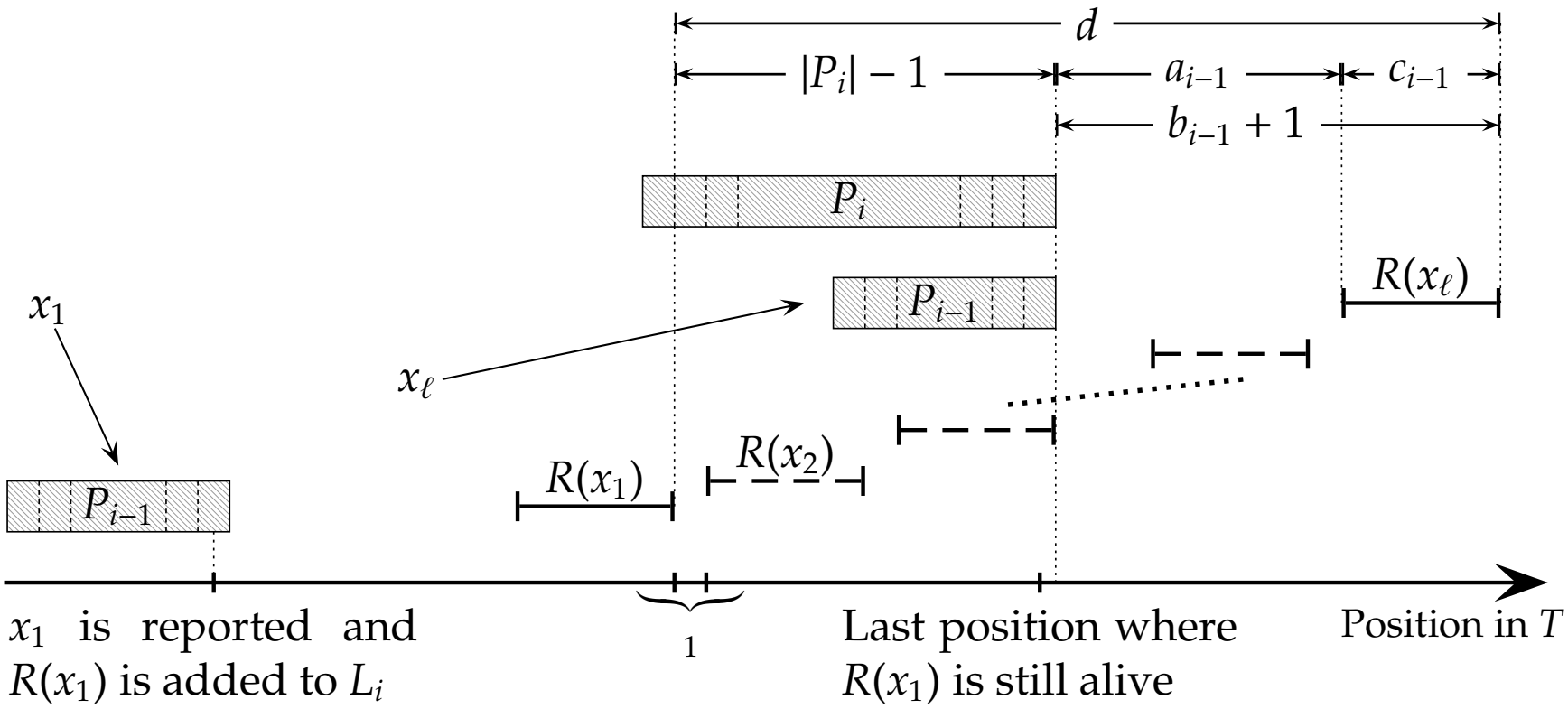

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Total space: $\displaystyle\sum_{i=2}^{k} |L_i| = O\left(\sum_{i=2}^{k} |P_i| + \sum_{i=1}^{k-1} a_i \right) = O(m + A)$